GRAFTON'S

GRADED ARITHMETIC

BOOK IV.

BY
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SUPERINTENDENT OF CITY SCHOOLS,
MONTREAL.

LOGARITHMS

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NOTE TO TEACHERS.

This book will be found to cover the requirements of both the preliminary and advanced A.A. examinations and of the examination for Model School diplomas.

The examples have been carefully graded, beginning with easy, straightforward exercises, and leading up to others which require some thought and ingenuity besides the mere accurate manipulation of figures and tables.

It is hoped that the sections on mensuration and logarithms will be found especially useful, not only for preparing students for examinations in which a knowledge of mensuration and logarithms is required, but also as furnishing an instructive sequel to the higher parts of arithmetic.

No Teachers' Manual accompanies this book, as teachers of advanced work generally prefer to use their own methods. Answers to the examples will be found at the end of the book.

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FRACTIONS.

A number is divisible by 2 if its *units digit* is divisible by 2.

A number is divisible by 3 if the *sum of its digits* is divisible by 3.

A number is divisible by 4 if the number expressed by its *two lowest digits* is divisible by 4.

A number is divisible by 5 if its units digit is 5 or 0.

A number is divisible by 8 if the number expressed by its *three lowest digits* is divisible by 8.

A number is divisible by 9 if the *sum of its digits* is divisible by 9.

A number is divisible by 11 if the difference between the sums of its digits in alternate places is either 0 or divisible by 11.

Tell at sight whether the following numbers are divisible by 2, 3, 4, 5, 6, 8, 9, 10 or 11:—

1. 288.	4. 715.	7. 4851.	10. 16500.
2. 960.	5. 648.	8. 3520.	11. 40040.
3. 912.	6. 495.	9. 2200.	12. 33165.

I.

Find the L.C.M. of :—

1. 7, 3, 56, 49, 21.
2. 35, 45, 55, 65.
3. 19, 38, 15, 20.
4. 12, 15, 16, 21.
5. 120, 144, 24, 96.
6. 10, 14, 15, 21, 28.
7. 22, 26, 30, 32, 39.
8. 96, 110, 120, 121.

Find by factoring the H.C.F. and L.C.M. of :—

9. 66, 99.
10. 56, 140.
11. 51, 68.
12. 45, 81.
13. 720, 840.
14. 37, 518.
15. 205, 287.
16. 230, 414.
17. 60, 75, 90.
18. 26, 52, 65.
19. 24, 60, 72, 108.
20. 19, 38, 57, 114.

Find the H.C.F. of :—

21. 14112 and 22176.
22. 18776 and 49287.
23. 735 and 1176.
24. 1065 and 16614.
25. 8874 and 17697.
26. 2079 and 33638.
27. 567, 1674 and 4041.
28. 354, 876, and 3645.

Reduce to lowest terms without using H.C.F. :—

29. $\frac{90}{105}$.
30. $\frac{99}{153}$.
31. $\frac{63}{117}$.
32. $\frac{330}{570}$.
33. $\frac{150}{225}$.
34. $\frac{212}{265}$.
35. $\frac{17}{153}$.
36. $\frac{729}{810}$.
37. $\frac{133}{161}$.
38. $\frac{39}{780}$.
39. $\frac{147}{2856}$.
40. $\frac{714}{918}$.
41. $\frac{763}{1414}$.
42. $\frac{111}{370}$.
43. $\frac{455}{1645}$.
44. $\frac{1775}{2350}$.
45. $\frac{925}{1025}$.
46. $\frac{483}{920}$.
47. $\frac{195000}{221000}$.
48. $\frac{1590}{1850}$.
49. $\frac{999}{1017}$.
50. $\frac{870}{1740}$.
51. $\frac{5750}{33500}$.
52. $\frac{11650}{15850}$.

Reduce to lowest terms, using H.C.F. :—

53. $\frac{247}{380}$.
54. $\frac{53}{265}$.
55. $\frac{791}{1017}$.
56. $\frac{1515}{1616}$.
57. $\frac{39139}{78339}$.
58. $\frac{103}{7519}$.
59. $\frac{591}{1576}$.
60. $\frac{1775}{1925}$.
61. $\frac{477}{750}$.
62. $\frac{703}{4107}$.
63. $\frac{4720}{90063}$.
64. $\frac{999}{2835}$.
65. $\frac{1197}{1477}$.
66. $\frac{79}{4819}$.
67. $\frac{8307}{72900}$.
68. $\frac{124223}{136653}$.
69. $\frac{13690}{99900}$.
70. $\frac{9664}{9696}$.

Define *prime factor*, H.C.F., L.C.M.

II.

MENTAL PRACTICE IN SIMPLE RULES.

(Where ½ or ¼ or ¾ occurs in the multiplicand, instead of adding two ciphers, add 50, or 25 or 75 respectively.)

1. Multiply by 5 :—
   76, 384, 9875, $67\frac{1}{2}$, $86\frac{3}{4}$, $91\frac{1}{4}$, $167\frac{1}{2}$, $869\frac{1}{4}$.
2. Divide by 5 :—
   16, 37, 84, 356, 685, 769, 876, $687\frac{1}{2}$, $769\frac{3}{4}$.
3. Multiply by 25 :—
   100, 350, 4750, 6845, $65\frac{1}{2}$, $84\frac{1}{4}$, $32\frac{3}{4}$, $76\frac{3}{4}$.
4. Divide by 25 :—
   75, 375, 875, 764, 879, 989, 2375, 5875.
5. Multiply by $12\frac{1}{2}$:—
   840, 845, 1845, 1840, $78\frac{1}{2}$, $91\frac{1}{4}$, $5\frac{3}{4}$, $91\frac{1}{2}$.
6. Divide by $12\frac{1}{2}$:—
   75, 750, 1250, 25, 125, 78, 85, 68, 79.
7. Multiply by 125 :—
   60, 56, 560, 565, 1565, $76\frac{1}{2}$, $91\frac{1}{4}$, $75\frac{3}{4}$.
8. Divide by 250 :—
   1250, 3450, 4750, 8975, 2525, 932, 6845.
9. 19×18; 11×36; 48×17; 49×19; 68×25; $7 \times \frac{1}{3}$ of 81; $8 \times \frac{1}{5}$ of 80; $9 \times \frac{1}{9}$ of 117; $12 \times \frac{2}{3}$ of itself; $168 \times \frac{1}{4}$ of 36; $8\frac{1}{2} \times 17$; $9\frac{1}{3} \times 28$; $8\frac{1}{3} \times 25$.

10. Give the prime factors of :—84, 85, 261, 117, 143, 176, 484, 210, 441, 378, 963, 1605, 292.

11. Find the H.C.F. of :—6, 12; 9, 15; 84, 91; 49, 259; 85, 51; 165, 300; 16, 84; 29, 261.

12. Find the L.C.M. of :—3, 4, 5; 6, 8, 10; 6, 10, 15; 7, 14, 21; 7, 11, 13; 9, 10, 12; 20, 16, 24; 25, 50, 75; 5, 6, 7; 7, 8, 9.

1. To add or subtract common fractions reduce them to similar fractions, and then add or subtract the numerators.

2. To multiply a common fraction by an integer, either multiply the numerator or divide the denominator. To divide a common fraction by an integer, either divide the numerator or multiply the denominator.

3. To multiply a common fraction by another common fraction, multiply the numerators to find the new numerator, and the denominators to find the new denominator.

4. To divide by a common fraction, invert the divisor and multiply.

5. To change the form of a common fraction without altering its value, multiply or divide both numerator and denominator by the same number.

III.

Find the value of :—

1. $\frac{4}{5}+\frac{6}{7}+\frac{12}{15}+\frac{8}{9}$.
2. $\frac{8}{9}+\frac{9}{10}+\frac{10}{11}+\frac{11}{12}$.
3. $5\frac{1}{6}+6\frac{1}{7}+7\frac{1}{8}+9\frac{11}{12}$.
4. $7\frac{10}{19}+\frac{19}{37}+4\frac{3}{95}+18\frac{11}{19}$.
5. $8\frac{1}{4}+56\frac{3}{5}+7\frac{1}{6}+374+12\frac{5}{8}+47\frac{4}{9}+98\frac{7}{10}+29$.
6. $68\frac{4}{15}+\frac{11}{16}+72+\frac{17}{24}+6\frac{1}{12}+\frac{3}{16}+19\frac{5}{8}+12\frac{4}{9}$.
7. $8-6\frac{3}{17}$.
8. $\frac{3}{5}-\frac{5}{16}$.
9. $50\frac{9}{13}-19$.
10. $8\frac{13}{96}-4\frac{12}{144}$.
11. $10\frac{1}{51}-6\frac{3}{34}$.
12. $13\frac{39}{50}-8\frac{74}{75}$.
13. $113\frac{85}{120}-56\frac{42}{112}$.
14. $161\frac{15}{120}-131\frac{31}{132}$.
15. $118\frac{2}{27}-\frac{7}{63}$.
16. $3-2\frac{9}{14}+\frac{1}{4}$.
17. $4\frac{7}{12}-3\frac{9}{16}+\frac{4}{5}$.
18. $8\frac{1}{3}-2\frac{2}{5}-3\frac{7}{16}$.
19. $\frac{1}{2}-\frac{1}{3}+\frac{7}{8}-\frac{5}{12}$.
20. $\frac{7}{8}+\frac{7}{12}+\frac{7}{16}-\frac{7}{18}$.
21. $\frac{19}{15}-\frac{8}{15}+\frac{1}{30}-\frac{13}{50}$.
22. $\frac{3}{16}+\frac{13}{15}-\frac{1}{5}+\frac{4}{9}$.
23. $10-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}$.
24. $5+3\frac{1}{4}-4+\frac{2}{7}-\frac{2}{9}$.
25. $\frac{3}{8}-6\frac{1}{3}+8\frac{1}{4}+\frac{5}{6}$.
26. $8\frac{1}{2}+\frac{2}{7}+\frac{5}{9}-3\frac{5}{6}$.
27. $7+\frac{1}{4}+\frac{1}{5}-\frac{1}{6}-\frac{1}{8}-\frac{1}{10}$.
28. $\frac{1}{2}+\frac{1}{3}-(\frac{1}{4}+\frac{1}{5})$.
29. $7-(1\frac{1}{4}+2\frac{1}{3})-(\frac{5}{6}-\frac{3}{4})$.

30. $\frac{4}{7} \times 1\frac{4}{15} \times \frac{7}{8} \times \frac{9}{16}$.

31. $\frac{9}{10} \times \frac{2}{3} \times \frac{5}{12} \times \frac{3}{8}$.

32. $\frac{11}{13} \times \frac{5}{9} \times \frac{3}{14} \times \frac{17}{19}$.

33. $6\frac{2}{3} \times 73\frac{1}{5} \times \frac{30}{61} \times 5$.

34. $11\frac{2}{3} \times \frac{1}{7} \times 9\frac{1}{10} \times 1\frac{2}{13}$.

35. $5\frac{1}{7} \times 1\frac{8}{9} \times 1\frac{6}{17} \times \frac{5}{7}$.

36. $1\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{1}{3} \times 6\frac{1}{2} \times \frac{8}{91}$.

37. $1\frac{1}{4}$ of $1\frac{1}{2} \times \frac{3}{2}$ of 2.

38. $31\frac{1}{12}$ of $\frac{5}{6} \times 51 \times \frac{1}{94}$.

39. $16\frac{1}{2} \times \frac{5}{8} \times 23\frac{2}{5} \times 11$.

40. $8\frac{1}{2} \times 1\frac{1}{3} \times 21\frac{1}{5} \times \frac{5}{34}$.

41. $\frac{5}{18}$ of $\frac{17}{20}$ of $36\frac{19}{30} \times \frac{25}{34}$.

42. $\frac{50}{65} \times 1\frac{5}{6} \times \frac{4}{5} \times \frac{35}{36} \times \frac{7}{15} \times 2\frac{2}{5} \times 1\frac{1}{7} \times 12\frac{3}{5} \times 21\frac{1}{4} \times \frac{1}{33}$.

43. $4\frac{1}{2} \times \frac{12}{25} \times 1\frac{17}{18} \times \frac{31}{32} \times \frac{9}{35} \times 1\frac{82}{93} \times \frac{9}{13} \times 1\frac{3}{7} \times 8\frac{2}{3}$.

44. $(\frac{2}{5} + \frac{3}{4} + \frac{5}{6}) \times \frac{4}{5}$.

45. $(\frac{3}{8} + \frac{5}{6}) \times (\frac{2}{7} + \frac{7}{12})$.

46. $(2\frac{1}{3}$ of $\frac{6}{7}) - (1\frac{1}{2}$ of $1\frac{1}{4})$.

47. $(1\frac{2}{3} \times \frac{11}{17}$ of $\frac{17}{22}) - \frac{4}{23}$.

48. $19\frac{1}{4}$ of $\frac{8}{11}$ of $3 - 15\frac{3}{4}$.

49. $5\frac{2}{3}$ of $\frac{16}{17} + \frac{8}{11}$ of $\frac{5}{6}$.

50. $21\frac{1}{12}$ of $\frac{5}{7}$ of $\frac{7}{9}$ of $3 - \frac{7}{36}$.

51. $(1\frac{1}{3} \times \frac{9}{10}$ of $7) - \frac{3}{5}$ of $5\frac{1}{2}$.

52. $28\frac{1}{3} \div 16\frac{4}{5}$.

53. $131\frac{1}{5} \div 3\frac{1}{23}$.

54. $17\frac{4}{7} \div 21\frac{2}{3}$.

55. $231\frac{9}{20} \div 14\frac{9}{16}$.

56. $1 \div 9\frac{1}{11}$.

57. $10\frac{5}{7} \div 1\frac{1}{14}$.

58. $5\frac{8}{11} \div \frac{34}{55}$.

59. $31\frac{3}{4} \div 37\frac{7}{6}$.

60. $14\frac{3}{8} \div 73$.

61. $3\frac{3}{4} \times \frac{1}{2} \div \frac{7}{8}$.

62. $\frac{1}{2} \times \frac{5}{7} \div 4$.

63. $\frac{1}{9}$ of $\frac{1}{2} \div 6\frac{1}{3}$.

64. $6\frac{1}{4}$ of $\frac{11}{12} \div 275$.

65. $19\frac{1}{3} \div \frac{1}{2}$ of $\frac{1}{3}$.

66. $11\frac{2}{9} \div \frac{3}{5}$ of $\frac{5}{9}$.

67. $1\frac{9}{10} \times \frac{7}{8}$ of $\frac{2}{21} \div \frac{6}{7}$.

68. $5\frac{1}{6}$ of $\frac{19}{95} \div \frac{57}{66}$.

69. $(\frac{1}{2} + \frac{2}{3}) \div 7$.

70. $(1\frac{3}{19} - 1\frac{1}{38}) \div 6$.

71. $(\frac{5}{7} - \frac{1}{3}) \div \frac{4}{7}$.

72. $(1\frac{1}{8} + 2\frac{11}{12}) \div \frac{6}{8}$.

73. $5\frac{1}{7}$ of $\frac{2}{9} \div (\frac{1}{2} + \frac{2}{3})$.

74. $(\frac{1}{4} + \frac{1}{5} + \frac{1}{12}) \div \frac{7}{8}$.

75. $(1\frac{1}{2} + \frac{1}{9}) \div \frac{7}{8}$ of 24.

76. $12\frac{1}{7} \div (19\frac{1}{6} + 7)$.

77. $(\frac{4}{7} \times \frac{5}{9}) \div (\frac{4}{7} - \frac{5}{9})$.

78. $(\frac{3}{5} \times \frac{4}{15}) \div (\frac{3}{5} + \frac{4}{15})$.

79. $(\frac{14}{15} - \frac{3}{10}) \div (\frac{5}{6} + \frac{1}{8} + \frac{7}{12})$.

80. $(\frac{17}{20} - \frac{4}{15}) \div (\frac{13}{18} - \frac{9}{16})$.

81. $(\frac{9}{14} - \frac{13}{21}) \div (\frac{4}{7} \times \frac{13}{15} \times \frac{7}{8})$.

82. $(\frac{17}{36} \div \frac{11}{30}) \div (\frac{39}{40} - \frac{23}{30})$.

83. $(\frac{29}{50} \div \frac{36}{37}) \div (\frac{43}{51} \div \frac{866}{1169})$.

84. $2\frac{1}{2} \div (1\frac{1}{4} \div 1\frac{7}{8} + 3\frac{3}{4})$.

85. $(5\frac{1}{6} \div \frac{29}{48})$ of $\frac{7}{8}$ of 3.

86. $(8\frac{1}{2} \div \frac{34}{7}) \div 2\frac{1}{6}$ of $\frac{12}{13}$.

87. $(1 \div \frac{2}{3}) \div (1\frac{1}{7}$ of $6)$.

88. $(7\frac{1}{6} + 2\frac{2}{3}) \div (\frac{6}{7} \times \frac{5}{9})$.

89. $1 \div \frac{7}{8}$ of $\frac{3}{4}$.

90. $11\frac{1}{3}$ of $\frac{9}{10} \div 1\frac{7}{18} \times \frac{4}{5}$.

91. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5} \div 1\frac{1}{7}$ of $\frac{7}{16}$.

92. $15\frac{7}{12} \div 1\frac{1}{12}$ of $1\frac{7}{15}$.

93. $7\frac{1}{3}$ of $5\frac{2}{7} \div \frac{37}{44}$.

IV.

Simplify:—

1. $\dfrac{10}{2\frac{3}{4}}$ 2. $\dfrac{12\frac{5}{9}}{7}$ 3. $\dfrac{9\frac{5}{8}}{\frac{11}{16}}$ 4. $\dfrac{\frac{17}{20}}{13\frac{3}{10}}$ 5. $\dfrac{6\frac{8}{11}}{\frac{19}{22}}$

6. $\dfrac{3\frac{1}{5}-2\frac{2}{5}}{4\frac{1}{10}-3\frac{4}{5}}$ 7. $\dfrac{8-3\frac{4}{5}}{4\frac{9}{10}\div 1\frac{5}{9}}$ 8. $\dfrac{3\frac{1}{3}-1\frac{1}{18}}{2\frac{1}{3}+2\frac{1}{4}}$

9. $\dfrac{7-3\frac{4}{5}}{1\frac{5}{7}\times 2\frac{2}{3}}$ 10. $\dfrac{4\frac{2}{9}}{1\frac{4}{15}}\div\dfrac{2\frac{17}{24}}{1\frac{7}{32}}$ 11. $\dfrac{1\frac{17}{25}}{1\frac{41}{45}}\times\dfrac{1\frac{4}{51}}{2\frac{31}{34}}$

12. $\frac{1}{3}$ of $3\frac{1}{2}-\dfrac{3}{4\frac{1}{2}}+\dfrac{1}{1\frac{1}{3}}$ 13. $\dfrac{7\frac{3}{7}}{1\frac{5}{7}}+7\frac{7}{8}\times\frac{24}{25}+\frac{4}{15}$

14. $\frac{2}{7}$ of $\dfrac{3\frac{4}{5}}{2\frac{5}{7}}+\frac{3}{8}$ of $2\frac{2}{7}$ 15. $\dfrac{3\frac{3}{8}}{6\frac{3}{4}}+\dfrac{1}{3\frac{1}{5}}+\frac{3}{20}$ of $1\frac{1}{4}$

16. $\dfrac{26\frac{3}{7}-1\frac{3}{21}}{\frac{4}{9}+1\frac{1}{5}-\frac{1}{3}}$ 17. $\dfrac{1\frac{5}{6}+\frac{3}{8}+2\frac{7}{12}-\frac{3}{16}}{3\frac{1}{12}\times 1\frac{1}{32}}$

18. $\dfrac{1\frac{7}{9}}{9\frac{9}{11}}\times\dfrac{\frac{27}{64}}{\frac{11}{12}}\times\dfrac{2\frac{5}{6}}{\frac{21}{100}}\times\dfrac{\frac{15}{34}}{4\frac{4}{7}}$ 19. $\dfrac{\frac{10}{3}+\frac{5}{6}-\frac{20}{21}}{\frac{5}{6}-\frac{4}{7}}$

20. $\dfrac{3\frac{1}{2}\times\frac{6}{21}\times\frac{5}{51}}{125\frac{1}{3}\div 17}$ 21. $\dfrac{2\frac{2}{3}\times\frac{5}{16}\times 7\frac{1}{2}-3\frac{1}{5}\times 1\frac{1}{4}}{13\frac{1}{2}\times\frac{6}{25}+2\frac{2}{3}\div 1\frac{3}{5}}$

22. $(\frac{5}{2}-\frac{2}{3})\div(\frac{4}{3}-\frac{3}{4})$. 23. $\dfrac{\frac{1}{9}\text{ of }1\frac{1}{4}\text{ of }4\frac{1}{2}}{\frac{5}{56}\text{ of }1\frac{1}{8}\text{ of }3\frac{1}{2}}$

24. $3+\dfrac{1\frac{3}{8}+\frac{11}{12}}{2\frac{11}{13}-1\frac{9}{20}}$ of $4\frac{1}{12}$ of $3\frac{3}{7}$.

25. $\dfrac{\frac{5}{14}-\frac{3}{7}\text{ of }\frac{1}{2}}{\frac{5}{16}+\frac{7}{12}\text{ of }3\frac{1}{4}-(\frac{7}{8}\text{ of }\frac{3}{21}-\frac{1}{3})}\div\dfrac{\frac{1}{3}\text{ of }\frac{1}{2}+\frac{3}{2}\text{ of }5}{9\frac{1}{3}-1\frac{2}{3}}$

26. $\dfrac{(\frac{1}{2}+\frac{1}{3})(\frac{1}{2}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{2}-\frac{1}{4})}-\dfrac{(\frac{1}{3}+\frac{1}{5})(\frac{1}{3}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{3})(\frac{1}{3}-\frac{1}{4})}+\dfrac{(\frac{1}{4}+\frac{1}{5})(\frac{1}{4}+\frac{1}{6})}{(\frac{1}{2}-\frac{1}{4})(\frac{1}{3}-\frac{1}{4})}$

V.

1. A post is $\frac{1}{6}$ in the ground, $\frac{4}{9}$ in the water, and 14 feet out of the water; find the length of the post.

2. One-sixteenth of a certain number exceeds $\frac{1}{20}$ of it by 11; find the number.

3. Find the number whose eighth part and twelfth part together make 10.

4. After giving away $\frac{2}{9}$, $\frac{5}{12}$ and $\frac{4}{15}$ of my money, I had \$51 left; how much had I at first?

5. Reduce $\frac{1}{2}\frac{2}{3}\frac{4}{2}\frac{2}{3}$ to its lowest terms.

6. What number, divided by $3\frac{2}{7}$, will give $8\frac{1}{3}$ for quotient?

7. Subtract half the sum of $\frac{2}{9}$ and $\frac{3}{5}$ from $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$.

8. After spending $\frac{8}{15}$ of my money, I lost $\frac{5}{6}$ of what remained, and then had \$28; how much had I at first?

9. If $\frac{1}{3}+\frac{1}{10}$ of a farm were worth \$50, what would half the remainder be worth?

10. Divide the sum of $3\frac{1}{4}$ and $2\frac{1}{3}$ by their difference.

11. Express $\frac{2}{3}$ of $\frac{5}{6}$ of 12 acres as a fraction of 17 square miles.

12. Find the difference between the sum and the product of $\frac{5}{8}$ and $2\frac{7}{12}$.

13. Difference between $\frac{4\frac{2}{3}}{\frac{8}{9}}$ and $2\frac{3}{8}$?

14. What number must be added to $4\frac{1}{4}$ that $\frac{1}{6}$ of the sum may be $47\frac{1}{4}$?

15. Express $3\frac{1}{4}$ inches as a fraction of $4\frac{1}{2}$ yds.

16. Take $\frac{1}{3}$ of $\frac{1}{7}$ of £12. 12s. from $\frac{2}{3}$ of $\frac{3}{4}$ of £5.

17. Divide the difference between $\frac{5}{12}$ and $\frac{3}{5}$ by the product of $1\frac{1}{2}$ and $\frac{5}{8}$.

18. I own $\frac{3}{4}$ of a ship and sell $\frac{8}{9}$ of my share for \$914; how much is the whole ship worth?

19. A yacht worth \$900 and insured for \$540 is lost; $\frac{1}{8}$ of the yacht belonged to A, $\frac{1}{4}$ to B and the rest to C. How much did C lose?

20. Add $23\frac{5}{8}$, $\frac{7\frac{3}{4}}{\frac{4}{9}}$ and $\frac{5}{7}$ of $3\frac{1}{2}$.

21. Find the difference between the sum and the product of $\frac{5}{8}$, $\frac{7}{10}$ and $\frac{8}{15}$.

22. If a certain number were added to $\frac{3}{4}$ of $12\frac{1}{8}$, the result would be equal to $\frac{7}{8}$ of $50\frac{1}{2}$; find the number.

23. Multiply the sum of $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{8}$, $\frac{7}{10}$ and $\frac{11}{12}$ by the difference between $1\frac{2}{3}$ and $4\frac{1}{2}$; and divide the product by $\frac{3\frac{4}{9}}{2\frac{3}{4}}$

24. Which is greater, $\frac{7}{12}$ of $7\frac{1}{8}$ or $\frac{5}{9}$ of $8\frac{3}{10}$, and by how much?

25. Which is greater, $\frac{3\frac{1}{2}}{\frac{5}{8}}$ or $\frac{7\frac{2}{3}}{1\frac{3}{4}}$ and by how much?

26. Divide the sum of $1\frac{1}{20}$ and $\frac{1}{1\frac{4}{9}}$ by their difference.

27. Multiply the sum of $\frac{4}{5}$, $\frac{5}{6}$, $\frac{3}{8}$ and $\frac{9}{10}$ by the difference between $\frac{11}{12}$ and $\frac{14}{15}$, and divide the product by $\frac{2}{3}$ of $\frac{15}{16}$ of $8\frac{3}{4}$.

28. What is the smallest increment which will convert $\frac{15}{16} + 11\frac{7}{12} + \frac{9\frac{3}{4}}{1\frac{5}{8}}$ into an integer?

29. Find the product of two numbers whose sum is $19\frac{5}{12}$ and difference $3\frac{7}{8}$.

30. Find two numbers whose sum and difference are $5\frac{2}{13}$ and $1\frac{7}{18}$ respectively; and divide the greater by the less.

31. If $\frac{2}{3}$ is worth \$270, what is the value of $\frac{3}{11}$?

32. If 3 be added to each term of the fraction $\frac{4}{5}$, is its value increased or diminished? by how much?

VI.

MENTAL PRACTICE IN COMMON FRACTIONS.

1. Reduce to improper fractions:—$6\frac{1}{2}$, $7\frac{1}{4}$, $8\frac{8}{11}$, $9\frac{1}{12}$, $17\frac{1}{3}$, $18\frac{2}{5}$, $19\frac{3}{4}$, $21\frac{7}{8}$, $76\frac{1}{2}$, $68\frac{3}{4}$, $111\frac{1}{9}$, $142\frac{6}{7}$.

2. Reduce to mixed numbers in lowest terms:—$\frac{18}{7}$, $\frac{118}{7}$, $\frac{106}{13}$, $\frac{211}{19}$, $\frac{846}{19}$, $\frac{365}{146}$, $\frac{2000}{1111}$, $\frac{143}{117}$, $\frac{441}{210}$, $\frac{378}{108}$, $\frac{1825}{1000}$.

3. Reduce to equivalents having a C.D.:—$\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$; $\frac{1}{3}$, $\frac{4}{6}$, $\frac{7}{9}$; $\frac{1}{8}$, $\frac{3}{16}$, $\frac{9}{24}$; $\frac{11}{12}$, $\frac{13}{15}$, $\frac{17}{20}$; $\frac{9}{11}$, $\frac{4}{22}$, $\frac{5}{33}$; $\frac{8}{9}$, $\frac{7}{24}$, $\frac{11}{36}$.

4. Find the sum of:—$\frac{6}{8}$, $\frac{7}{8}$, $\frac{5}{8}$, $\frac{11}{8}$; $\frac{1}{7}$, $\frac{3}{7}$, $2\frac{1}{7}$, $3\frac{3}{7}$; $\frac{11}{14}$, $\frac{9}{14}$, $\frac{7}{14}$, $1\frac{1}{14}$; $\frac{8}{9}$, $\frac{8}{9}$ of $\frac{8}{9}$; $\frac{8}{9}$, $\frac{8}{9}$ of $\frac{8}{9}$; $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$; $\frac{7}{11}$, $\frac{1}{12}$; $\frac{1}{12}$, $\frac{1}{14}$; $\frac{2}{9}$, $\frac{5}{27}$; $\frac{2}{9}$, $\frac{3}{18}$, $\frac{5}{27}$; $\frac{6}{15}$, $\frac{6}{30}$, $\frac{6}{45}$, $\frac{6}{75}$; $7\frac{1}{2}$, $8\frac{1}{2}$, $9\frac{1}{2}$; $16\frac{1}{4}$, $18\frac{1}{2}$, $20\frac{3}{4}$; $17\frac{1}{5}$, $22\frac{2}{5}$, $40\frac{3}{5}$.

5. Find the difference between:—$\frac{1}{8}$, $\frac{1}{11}$; $\frac{1}{11}$, $\frac{1}{18}$; $\frac{1}{18}$, $\frac{1}{27}$; $\frac{1}{27}$, $\frac{1}{81}$; $\frac{2}{3}$, $\frac{2}{5}$; $\frac{2}{7}$, $\frac{2}{17}$; $\frac{3}{14}$, $\frac{3}{19}$; $\frac{19}{20}$, $\frac{18}{19}$; $7\frac{1}{2}$, $3\frac{1}{3}$; $6\frac{3}{4}$, $4\frac{1}{2}$; $11\frac{9}{10}$, $9\frac{10}{11}$; 54, $16\frac{4}{13}$; $29\frac{8}{17}$, 14.

6. Value of:—$\frac{1}{3}\times\frac{7}{8}\times\frac{9}{10}$; $2\frac{1}{2}\times\frac{2}{3}\times\frac{7}{9}$; $\frac{16}{17}\times\frac{17}{18}\times 2\frac{2}{3}$; $4\frac{1}{2}\times 6\frac{1}{3}\times\frac{6}{117}$; $(\frac{13}{14}-\frac{1}{2})\times 14$; $\frac{3}{8}$ of $1\frac{1}{2}$ of $\frac{7}{16}$; $\frac{11}{22}$ of $\frac{16}{64}$ of 16; $\frac{6\times 8\times 3}{12\times 16}$, $\frac{18\times 16\times 14}{56\times 36}$, $\frac{4\times 9\times 3\times 7\times 3}{63\times 36}$, $\frac{121\times 169}{143\times 143}$.

7. Value of:—$\frac{2}{3}\div\frac{1}{3}$; $\frac{1}{3}\div\frac{2}{3}$; $5\div\frac{3}{2}$; $\frac{3}{2}\div 5$; $6\frac{1}{2}\div 2\frac{1}{6}$; $2\frac{1}{6}\div 6\frac{1}{2}$; $7\frac{1}{3}\div 8\frac{1}{3}$; $\frac{14}{15}\div\frac{1}{2}$ of $\frac{3}{7}$; $\frac{1}{3}$ of $\frac{2}{7}\div\frac{20}{30}$.

8. Reduce:—\$.55 to the fraction of \$1; 12s. 6d. to fraction of £1; $7\frac{1}{2}$d. to fraction of 1s.; 1 hr. 30 min. to fraction of a day; 3 qts. 1 pt. to fraction of a gal.; 1 yd. 1 ft. to a fraction of a mile; 4 lbs. 8 oz. to fraction of a cwt.; 3 oz. 5 dwt. to fraction of a lb.

9. Value of:—$\frac{4}{7}$ of \$1; $\frac{4}{7}$ of 1s.; $\frac{5}{12}$ of \$3; $\frac{5}{12}$ of 2s. 6d.; $\frac{7}{8}$ of a ton; $\frac{7}{8}$ of a mile; $\frac{7}{10}$ of a lb. Av.; $\frac{7}{10}$ of a lb. Troy.

10. $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{1}{2}-\frac{1}{3}}$; $\frac{\frac{1}{2}\times\frac{1}{3}}{\frac{1}{2}\div\frac{1}{3}}$; $\frac{7\frac{1}{2}+8\frac{1}{2}}{8\frac{1}{2}-7\frac{1}{2}}$; $\frac{18\frac{1}{3}+9\frac{2}{3}}{4\frac{2}{3}\times 1\times\frac{12}{13}}$; $\frac{1}{3}$ of $16-\frac{1}{6}$ of 13; $\frac{1}{6}$ of $63\frac{1}{3}$; $26\frac{3}{4}+(4\frac{1}{4}-2\frac{1}{2})+3\frac{1}{2}$.

VII.

Reduce to decimals :—

1. $\frac{1}{3}$.
2. $\frac{4}{9}$.
3. $\frac{7}{12}$.
4. $\frac{4}{11}$.
5. $\frac{2}{15}$.
6. $\frac{5}{7}$.
7. $\frac{1}{9}$.
8. $\frac{1}{90}$.
9. $\frac{1}{900}$.
10. $\frac{1}{99}$.
11. $\frac{1}{9999}$.
12. $\frac{7}{300}$.
13. $5\frac{3}{13}$.
14. $7\frac{7}{18}$.
15. $6\frac{17}{24}$.
16. $2\frac{1}{27}$.
17. $1\frac{15}{22}$.
18. $20\frac{5}{36}$.
19. $\frac{4}{37}$.
20. $\frac{5121}{117}$.
21. $\frac{31}{9}$.
22. $\frac{16}{15}$.
23. $\frac{704}{27}$.
24. $\frac{821}{99}$.
25. $\frac{102}{101}$.
26. $\frac{53}{168}$.
27. $6\frac{4}{70}$.
28. $\frac{111}{220}$.
29. $\frac{85}{2835}$.
30. $\frac{37}{55}$.

Express as vulgar fractions in their lowest terms :—

(Examples 31–50 at sight.)

31. $\cdot\dot{1}$.
32. $\cdot0\dot{1}$.
33. $\cdot\dot{3}$.
34. $\cdot\dot{6}$.
35. $\cdot0\dot{3}$.
36. $\cdot\dot{2}\dot{3}$.
37. $\cdot\dot{1}\dot{3}$.
38. $\cdot0\dot{9}$.
39. $\cdot\dot{8}\dot{1}$.
40. $\cdot00\dot{9}$.
41. $\cdot0\dot{6}$.
42. $\cdot0\dot{5}$.
43. $\cdot00\dot{5}$.
44. $\cdot00\dot{5}$.
45. $\cdot00\dot{5}$.
46. $\cdot\dot{2}\dot{4}$.
47. $\cdot\dot{2}\dot{4}$.
48. $\cdot0\dot{1}$.
49. $\cdot0\dot{1}$.
50. $\cdot00\dot{1}$.
51. $3\cdot60\dot{8}$.
52. $3\cdot6\dot{0}\dot{8}$.
53. $3\cdot60\dot{8}$.
54. $10\cdot68\dot{1}$.
55. $3\cdot02083$.
56. $5\cdot\dot{2}30769$.
57. $4\cdot\dot{7}14285$.
58. $\cdot31\dot{2}0\dot{0}$.
59. $\cdot00\dot{1}2\dot{6}$.
60. $11\cdot0694$.
61. $20\cdot7083$.
62. $\cdot00055$.
63. $\cdot036\dot{1}$.
64. $21\cdot\dot{0}0\dot{3}$.
65. $5\cdot4\dot{1}\dot{6}$.
66. $17\cdot0\dot{6}9$.
67. $6\cdot3\dot{8}\dot{2}$.
68. $8\cdot0\dot{4}\dot{3}$.
69. $17\cdot3\dot{1}6$.
70. $2\cdot00\dot{5}$.

Express at sight as non-recurring decimals :—

($\cdot\dot{9}=1$, and may be replaced by 1 in the next place up.)

71. $\cdot\dot{9}$.
72. $\cdot0\dot{9}$.
73. $5\cdot\dot{9}$.
74. $\cdot00\dot{9}$.
75. $\cdot4\dot{9}$.
76. $\cdot02\dot{9}$.
77. $\cdot6189$.
78. $\cdot619\dot{9}$.
79. $\cdot3\dot{9}\dot{9}$.
80. $3\cdot\dot{9}99$.

VIII.

Find the value correct to 5 decimal places of :—

1. $7\cdot5+6\cdot8\dot{1}+\cdot\dot{9}0\dot{8}+\cdot2134$.
2. $7\cdot\dot{9}0+\cdot3\dot{4}16+3\cdot2\dot{4}5+1\cdot\dot{8}$.
3. $11+7\cdot2+\cdot08\dot{1}4+\cdot00\dot{2}\dot{1}$.
4. $\cdot\dot{1}-\cdot0\dot{9}$.
5. $1\cdot1\dot{3}-\cdot5\dot{8}7463$.
6. $1\cdot2-1\cdot\dot{1}76\dot{9}$.

7. $\cdot 3\dot{1}\dot{3} + 9\cdot 0\dot{6} + 7\cdot 0\dot{3}\dot{1} + \cdot 00\dot{7}\dot{1}\dot{4}$.

8. $\cdot 8\dot{2}9\dot{6}0\dot{3} + \cdot 5\dot{6}\dot{3}\dot{2} + 59\cdot 0\dot{3}\dot{7} + \cdot 0\dot{6}9\dot{2}$.

9. $\cdot 5\dot{1}\dot{6}\dot{7} - \cdot 2\dot{8}\dot{6}3\dot{4}$.

10. $\cdot 6\dot{0}3\dot{2} - \cdot 0\dot{7}8\dot{5}19\dot{6}$.

11. $\cdot 0\dot{3} \times \cdot 5\dot{8}$.

12. $\cdot 0\dot{0}9\dot{1} \times \cdot 7\dot{1}\dot{6}\dot{3}$.

13. $\cdot 5\dot{9}0\dot{6} \times \cdot 0\dot{7}$.

14. $1\cdot 2\dot{8}\dot{4} \times \cdot 0\dot{3}0\dot{7}$.

15. $\cdot \dot{7}14285 \times \cdot 36\dot{1}$.

16. $11\cdot 07\dot{2} \times 5\cdot 0\dot{8}\dot{6}$.

17. $\cdot 11\dot{2}1\dot{6} \times \cdot 00\dot{3}\dot{7}$.

18. $\cdot 004\dot{1} \times \cdot \dot{7}2\dot{5}$.

19. $3\cdot 0\dot{3} \div \cdot 5\dot{8}$.

20. $1\cdot 2\dot{7} \div \cdot 0\dot{3}\dot{7}$.

21. $\cdot 03\dot{1}4\dot{2} \div \cdot 0\dot{6}\dot{7}$.

22. $2\cdot 12\dot{4} \div \cdot 30\dot{2}$.

23. $\cdot 026 \div \cdot 7\dot{8}9\dot{5}$.

24. $\cdot \dot{3} \div \cdot 11\dot{5}\dot{6}$.

25. $\cdot 2\dot{0}\dot{7} \div 5\cdot 2\dot{9}\dot{4}$.

26. $11\cdot 06\dot{3} \div 3\cdot 2\dot{1}$.

Find the recurring decimal which is equal to

27. $\cdot 9\dot{3}\dot{6} + \cdot \dot{7}\dot{1}$.

28. $\cdot 7\dot{3}\dot{1}\dot{2} + \cdot 8\dot{9}$.

29. $\cdot 4187 + \cdot 30\dot{6} + \cdot 12\dot{5}$.

30. $4\cdot \dot{6} + \cdot 2\dot{5}\dot{1} + \cdot 02\dot{5}\dot{1}\dot{4}$.

31. $2\cdot 001 + \cdot 1\dot{8}\dot{1}\dot{8} + \cdot \dot{5}$.

32. $6\cdot \dot{6} - 4\cdot \dot{8}$.

33. $4\cdot 3\dot{5} - 2\cdot \dot{7}$.

34. $2\cdot 4\dot{5} - \cdot 0\dot{8}$.

35. $5\cdot 3\dot{1}\dot{4} - 4\cdot 6\dot{7}$.

36. $11\cdot 2\dot{1}\dot{3} - 4\cdot 6\dot{8}\dot{9}$.

37. $\cdot 59\dot{7}\dot{5} \times 18$.

38. $75\cdot 19\dot{7} \times 5\cdot 2$.

39. $9\cdot \dot{7} \times 2\cdot 4\dot{5}$.

40. $3\cdot \dot{6} \times 4\cdot 0\dot{9}$.

41. $3\cdot 2\dot{7} \times 12\cdot 8\dot{3}$.

42. $\cdot 6\dot{1}\dot{7} \div 16$.

43. $5\cdot \dot{8} \div 7\cdot 0\dot{6}$.

44. $7\cdot \dot{3} \div 2\cdot 9\dot{3}$.

45. $3\cdot 1\dot{8} \div 1\cdot 13\dot{6}$.

46. $\cdot 75\dot{8} \div 4\frac{1}{\dot{3}}$.

IX.

Find the value in compound quantities of :—

1. £7·8̇3̇.
2. 2·13̇9̇3̇ days.
3. 256·07̇3̇ yds.
4. 4·4̇5̇ miles.
5. ·08̇3̇ of an hour.
6. 5·2578125 of 8 wks.
7. 3·1̇ of 90°.
8. ·0̇6̇ of a rod.
9. 6·878 of 2 sq. yds. 2 sq. ft.
10. 5·8̇3̇ of 1 lb. 8 oz. (Troy.)
11. ·706 of 5 tons 11 cwt.
12. ·3865 of $7\frac{1}{2}$ sq. yds.
13. 3·998 of 1 yd. 1 ft. 6 in.
14. 3·75 of 65 rods.
15. ·0764 of $9\frac{3}{4}$ days.
16. 15·7196 of $3\frac{3}{4}$ miles.

17. $\dot{6}$ of £4. 4s. 9d. + $\dot{3}\dot{6}$ of £2. 5s. 10d.
18. $0\dot{7}\dot{3}$ of a day + 3·75 hours.
19. $\dot{3}$ of a yd. + $\dot{3}$ of a ft. + ·125 of 1 ft. 4 in.
20. $\cdot\dot{7}14285$ of a week − $\dot{6}$ of an hour.
21. $\dot{0}\dot{9}$ of 40 rods + $\cdot01\dot{3}\dot{6}$ of a mile.
22. $0\dot{3}$ of 1 lb. Troy + $\cdot4\dot{1}\dot{6}$ of 1 oz. Troy.
23. $\cdot07\dot{1}\dot{6}$ of 554·4 tons + $\cdot\dot{4}2857\dot{1}$ of 56 lbs.

Reduce :—

24. 3s. $4\frac{1}{2}$d. to the decimal of 19s. 6d.
25. 2 tons 3 cwt. to the decimal of 10 tons.
26. 3 oz. 2 dwts. to the decimal of a lb. Troy
27. 3 yds. 1 ft. to the decimal of $\frac{1}{8}$ mile.
28. 2 ft. 6 in. to the decimal of a yard.
29. 12 cwt. 56 lbs. to the decimal of 4 cwt. 56 lbs.
30. $2\frac{3}{4}$ pks. to the decimal of 3 bushels.
31. $7\frac{1}{2}$ gal. to the decimal of 18 gallons.
32. 5° 13′ 40″ to the decimal of 12° 30′.
33. $12\frac{3}{4}$ ac. to the decimal of 121 sq. rods.
34. 29 ac. 120 sq. rods to the decimal of a sq. mile.
35. $11\frac{3}{4}$ sq. ft. to the decimal of 4 sq. yds.

X.

1. Simplify $\dfrac{5\cdot\dot{8}-4\cdot9\dot{1}\dot{6}}{5\cdot375-2\cdot\dot{9}\dot{4}}$
2. Simplify $\dfrac{2\cdot6 \text{ of } \cdot\dot{8}\dot{1}}{3\cdot\dot{7}14285\dot{} \div 4\cdot125}$
3. Simplify $\dfrac{6\cdot75-1\cdot\dot{8}}{\cdot58\dot{3} \text{ of } 1\cdot\dot{6}}$
4. Simplify $\dfrac{\frac{8}{9} \text{ of } \frac{4}{9}}{\cdot15+\frac{3}{8}}$
5. How many pence in $\cdot58\dot{3}$ of a shilling ?
6. How many cwt. in $\cdot64\dot{9}$ of a ton ?

7. Express 42 rods as the decimal of half an acre.

8. After spending $\frac{5}{7}$ and ·125 of my income, I am able to save \$117 a year; what is my annual income?

9. Express a day as the decimal of a year.

10. Express as a vulgar fraction the difference between $\dot{0}0\dot{3}$ and $0\dot{0}3$.

11. What fraction having 24 for a denominator is equivalent to ·625?

12. Find by vulgar fractions sum of ·98, $\cdot\dot{9}\dot{8}$ and $\cdot 9\dot{8}$.

13. What fraction having 27 for numerator is equivalent to ·09375?

14. Express in lbs. the difference between ·034375 of a ton and ·90625 of a cwt.

15. Value of $\cdot 03\dot{2}\dot{4}$ of a mile.

16. Divide 1·02 by $\frac{17}{24}$ of ·144.

17. Which is the greater, $\cdot 7\dot{6}\dot{5}$ or $\cdot 76\dot{5}$, and by how much?

18. What number multiplied by the sum of $\cdot 6\dot{5}\dot{4}$ and $\cdot 6\dot{5}\dot{4}$ will give 1 for the product?

19. How many times is $9\cdot 0\dot{3}\dot{7}$ sq. rods contained in 244 ac.?

20. Simplify $\dfrac{\cdot 00281 \times \cdot 0625}{1\cdot 405}$

21. Simplify $(\cdot 5 + \cdot 75) \times (2\cdot 5 - \cdot 4) \div (125 + \frac{1}{4\cdot 8})$.

22. Simplify $\dfrac{2\cdot 7916 \times 3\cdot 23\dot{7}}{1\cdot 86\dot{1} \times \cdot 8093\dot{4}}$

23. Subtract ·0523 of 11 weeks from ·932 of 6 days, and give the answer in minutes.

24. Simplify $\dfrac{\frac{1}{3} \text{ of } \cdot\dot{3}}{\frac{1}{9} \text{ of } \cdot 1} + \dfrac{9\cdot 25}{\cdot 00925}$

25. Simplify $\dfrac{9\cdot 75 \text{ of } 3\cdot 6}{\cdot 039} + \dfrac{3\cdot 75}{\cdot 015}$

26. Express in yards 1·03125 mi. − 292·5 rods.

XI.

MENTAL PRACTICE IN DECIMALS.

1. Reduce to decimals:—$\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, $\frac{3}{8}$, $\frac{5}{8}$, $\frac{7}{8}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{7}{16}$, $\frac{13}{16}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{7}$, $\frac{1}{9}$, $\frac{18}{54}$, $\frac{19}{57}$, $\frac{64}{256}$, $\frac{11}{40}$.

2. Reduce to vulgar fractions:—·5, ·25, ·75, ·125, ·625, ·875, $\dot{3}$, ·33, ·6, ·606, ·66, ·78, ·11, ·$\dot{1}\dot{1}$, ·11, ·864, ·$\dot{8}\dot{6}\dot{4}$.

3. Add:—6, ·6, ·06, ·006; 67, ·67, 6·7; 16, 1·6, ·016; 834, 8·34, 83·4; 7·6, 8·4, 9·3; 8·5, ·15, ·65; $\frac{1}{2}$, $\frac{1}{4}$, ·8.

4. Find the difference between:—·$\dot{6}$ and ·6; ·$\dot{7}$ and ·7; ·18 and ·18; ·$\dot{0}\dot{6}$ and ·06; 78·91 and 36·74; 81·17 and 34·34.

5. Multiply:—6·8 × 5; 7·4 × 2·5; 68 × ·75; 7·5 × 7·5; ·66 × ·6; 80 × ·125; 72 × ·625; 78 × ·13; 3·$\dot{3}$ × ·6 × ·75.

6. Divide:—6·75 ÷ 5; 6·75 ÷ ·5; 32 ÷ ·4; 25 ÷ ·05; 27 ÷ ·009; ·056 ÷ ·7; ·4 ÷ 1·6; ·006 ÷ ·015; 22·5 ÷ ·09; ·0451 ÷ ·11; ·0049 ÷ ·035; 1·$\dot{7}$ ÷ 5; 4·6$\dot{4}$ ÷ 20.

7. Value of:—3·775 of \$10; ·1235 of \$1000; 4·625 of 4 cwt.; 3·285 of 4 mi.; 1·0325 tons; 13·775 cwt.; 8s. ÷ 9·6; ·75 of a lb.; ·$\dot{7}\dot{2}$ of a rod; ·375 of a lb. Troy.

8. Reduce:—4 hrs. to decimal of a day; 2s. 7½d. to decimal of a guinea; 3 pints to decimal of a gal.; 3 oz. to decimal of a lb. Troy; 4 in. to decimal of a yard; 4½ in. to decimal of a foot; 330 yds. to decimal of a mile.

9. One-tenth + one-hundredth + one-thousandth.
7-tenths + 92-hundredths + one-millionth.
7-tenths − 7-thousandths.
17 − 17-tenths + 17-ten-thousandths.

10. $\frac{1}{4}$ + ·2 + $\frac{1}{6}$ + 1$\frac{1}{6}$; 3$\frac{1}{8}$ − 3·125; 67·8 × $\frac{1}{3}$ of $\frac{3}{4}$; 16 ÷ $\frac{1}{3}$ of itself; \$$\frac{1}{2}$ + \$·33$\frac{1}{3}$ + \$$\frac{1}{3}$; 6$\frac{1}{2}$ + 7·75 + 8$\frac{3}{4}$; 10·8 ÷ 3$\frac{1}{3}$; ·1 ÷ 8$\frac{1}{7}$; 1 × ·33$\frac{1}{3}$ ÷ 2; 1·5 × 1·25; $\frac{1}{3}$ of 3 × ·$\dot{3}$ × $\frac{1}{3}^{1}$; ·$\dot{9}$ × ·9 × 1$\frac{1}{9}$; ·3 × ·6 × ·$\dot{7}$; \$15 ÷ 2·25; \$120 ÷ 1·44; \$·5 ÷ 2·5.

XII.

RATIO AND PROPORTION.

The relative magnitude of two numbers is called their **ratio.** Ratio is expressed by the fraction which the first number is of the second.

The first term of a ratio is called the **antecedent,** and the second term the **consequent.**

When two *ratios* are equal, the four terms are said to be in **proportion,** and are called **proportionals.**

The first and last terms of a proportion are called the **extremes,** and the two middle terms are called the **means.**

Test of a proportion.—Four numbers are proportionals when *the product of the extremes = the product of the means.*

If any three terms of a proportion are given, the missing term may be found, for *the product of the extremes divided by either mean gives the other mean; and the product of the means divided by either extreme gives the other extreme.*

Find at sight the ratio of the following:—

1. 21 : 7.	**7.** $\frac{2}{3} : \frac{7}{9}$.	**13.** 1 gal. : 1 qt.
2. 7 : 21.	**8.** $\frac{5}{8} : \frac{15}{16}$.	**14.** 30 rods : 1 acre.
3. 20 : 5.	**9.** $\frac{6}{7} : \frac{9}{14}$.	**15.** 1 c. yd. : 15 c. ft.
4. $2\frac{1}{2} : 10$.	**10.** $\cdot 2 : \frac{3}{10}$.	**16.** 4 yds. : 9 in.
5. $7 : 2\frac{1}{3}$.	**11.** $\cdot 5 : 2$.	**17.** 6 dys. : 16 hrs.
6. $2\frac{1}{2} : 7\frac{1}{2}$.	**12.** $3\frac{1}{3} : 50$.	**18.** 2s. 6d. : £1.

19. Is the ratio always an *abstract* number?

20. What is the effect of multiplying the antecedent? the consequent? Of dividing the antecedent? the consequent? (Show by examples.)

21. What is the effect of multiplying or dividing both terms by the same number? (Show by examples.)

Find the missing term in the following proportions:—

(Examples 22–36 at sight.)

22. $x : 5 :: 6 : 10$.

23. $7 : x :: 21 : 12$.

24. $5 : 9 :: x : 18$.

25. $6 : 11 :: 24 : x$.

26. $8 : 13 :: x : 65$.

27. $4 : x :: 16 : 68$.

28. $14 : 23 :: 42 : x$.

29. $26 : x :: 78 : 54$.

30. $32 : 20 :: 8 : x$.

31. $x : 50 :: 9 : 10$.

32. $51 : 27 :: x : 9$.

33. $63 : x :: 9 : 7$.

34. $3 : x :: 4 : 11$.

35. $x : 17 :: 5 : 9$.

36. $12 : 7 :: 8 : x$.

37. $x : 6 :: 31 : 18$.

38. $\frac{2}{3} : x :: 4\frac{1}{2} : 17$.

39. $\frac{3}{4} : 5\frac{1}{6} :: 13 : x$.

40. $8\frac{1}{4} : 7\frac{4}{9} :: x : \frac{5}{12}$.

41. $8\frac{3}{4} : 9\frac{1}{3} :: 10\frac{5}{6} : x$.

42. $\cdot089 : x :: 2\cdot4 : \cdot0984$.

43. $1\cdot333 : \cdot172 :: x : \cdot012$.

44. $4\cdot59 : 10\cdot8 :: \cdot00306 : x$.

45. $1\cdot02 : 5\cdot1 :: 1\cdot03 : x$.

46. $x : \frac{3}{4}$ of $\frac{8}{9} :: 21\frac{3}{7} : \frac{5}{18}$.

47. $\frac{2}{5}$ of $4\frac{1}{2} : x :: 6\frac{5}{8} : \frac{13}{15}$.

48. $1590 : 53 :: 1710 : x$.

49. $15\frac{1}{3} : 12 :: 46 : x$.

50. $2\cdot07 : \cdot051 :: \cdot69 : x$.

51. $17\cdot15 : 6\cdot32 :: 1\cdot87 : x$.

52. £1. 10s. : £1. 15s. :: £10. : x.

53. 5 dys. 3 hrs. : 61 dys. 12 hrs. :: \$914 : x.

54. $11\frac{1}{10}$ inches : $55\frac{1}{2}$ yards :: \$54 : x.

55. 18 bus. 3 pks. : 2116 bus. 1 pk. :: \$75 : x.

56. $19\frac{3}{4}$ ac. : 11 ac. 60 sq. rods :: 79 tons : x.

57. $60° 37' : 360° :: 151$ dys. 13 hrs. : x.

58. x : 3 qts. 1 pt. :: \$4 : \$0·10.

59. 8 oz. 6 dwt. : 13 dwt. 20 grs. :: x : \$0.44.

60. 34 dys. 16 hrs. : 17 hrs. 20 min. :: \$3.60 : x.

61. Assuming the earth's circumference to be 25,000 miles find its diameter, the ratio of the diameter to the circumference being 113 to 355.

62. Find a fourth proportional to (*a*) the sum, (*b*) the difference and (*c*) the product of $\frac{3}{4}$ and $\frac{5}{6}$.

63. What would 225 yards of cloth cost at the rate of £1. 17s. 6d. for $7\frac{1}{2}$ yds.?

64. If a loaf of bread weighs 1 lb. 4 oz. when flour is \$6.50 a barrel, what should a loaf of the same price weigh when flour is \$4.25 a barrel?

65. A workman digs out $\frac{2}{9}$ of a cubic yard of earth in $\frac{1}{4}$ hr.; how long would he be occupied in excavating a cellar 5 yds. square and 5 yds. deep?

66. If the first-class railway fare from Montreal to Quebec (180 m.) is \$4.50, what should be paid from Montreal to Toronto (333 miles)?

67. A cubic foot of water weighs $62\frac{1}{2}$ lbs.; what weight would a vessel 6 in. long, wide and deep contain?

68. Pure lead is 11·3 times heavier than water; find the weight of a block of lead 2 ft. 6 in. long, 2 ft. 5 in. broad and 18 in. thick.

69. A train travels $7\frac{1}{2}$ m. in 12·58 min.; how far will it travel in 5 hours?

70. Find the yearly wages of a coachman who was paid \$78 for services from 10th April to 14th June.

71. A clerk was engaged on 5th March at a salary of $\$574\frac{7}{8}$ a year. When he left he was paid $\$118\frac{1}{8}$; on what day did he leave?

72. If I lent a friend \$350 for 52 days, how long ought he in return to lend me \$280?

73. On 20th Apr. a friend lent me \$560 until 6th Aug. I repaid the debt by lending him a certain sum from 11th Oct. to 3rd Jan. How much did I lend him?

74. 3000 soldiers were supplied with provisions for 87 days; after 26 days so many men left that the provisions lasted 300 days more. How many men went away?

75. Find a fourth proportional to $\cdot\dot{9}\dot{8}$, $\cdot 9\dot{8}$ and $\cdot 98$.

76. Four-fifteenths of a ship's crew are able to do a piece of work in 22 days; how long would one-third of the remainder of the crew take to do it?

XIII.

PROPORTIONAL PARTS.

Divide at sight :—

1. \$48 into parts having the ratio 3 : 5.
2. \$60 into parts having the ratio 2 : 13.
3. \$25 into parts having the ratio 2 : 3.
4. \$44 into parts having the ratio 7 : 4.
5. \$80 into parts proportional to the numbers 5, 7 and 8.
6. \$28 into parts proportional to the numbers 4, 3 and 7.
7. \$10 into parts proportional to $\frac{1}{2}$ and $\frac{1}{3}$.
8. \$64 into parts proportional to $\frac{1}{3}$ and $\frac{1}{5}$.
9. \$15 into parts proportional to ·5 and ·75.
10. \$60 into parts having the ratios: (*a*) 2 : 3; (*b*) 5 : 1 : 4; (*c*) 7 : 8; (*d*) $\frac{1}{2} : \frac{3}{4}$; (*e*) 11 : 9 : 10; (*f*) ·7 : ·3.
11. Divide \$140 among 3 persons in the proportion of (*a*) 5 : 7 : 8; (*b*) 10 : 11 : 14; (*c*) $\frac{1}{6} : \frac{1}{3} : \frac{2}{3}$.
12. Divide \$66 between A and B so that
    (*a*) A shall have \$14 less than B.
    (*b*) A shall have \$20 more than B.
    (*c*) A shall have \$5 more than twice as much as B.
    (*d*) If A had \$7 more he would have twice as much as B.
13. Divide \$45 amongst A, B and C so that
    (*a*) A shall have twice as much as B, and C \$5 more than B.
    (*b*) A shall have twice as much as B, and C \$6 less than B.
    (*c*) A shall have \$10 more than B, and C half as much as B.
    (*d*) A shall have \$5 more than twice B's share and C half of A's share.

14. Divide \$105 among 4 men, 7 women and 5 children, so that a man may have as much as a woman and child together, and a women three times as much as a child.

15. Divide \$155 among 8 men, 2 women and 10 children, so that a man has half as much again as a woman, and a woman half as much again as a child.

16. Two persons contribute respectively \$1610 and \$1150. If the profits are \$852, what should each receive?

17. Two copyists are employed in transcribing a manuscript, and they copy respectively 224 and 288 pages. If \$140 is paid for the work, what should each receive?

18. In building a wall one mason works $\frac{4}{5}$ of the time, another $\frac{3}{4}$, and a third full time. What ought each to receive out of \$340?

19. If 8 men or 12 boys can do a piece of work in 20 days, in what time can 12 men and 8 boys do it?

20. How long would 15 boys be doing a piece of work which 8 men can do in 21 days, the work of 7 men being equivalent to that of 10 boys?

21. Divide 678 into three parts, such that the second shall be $\frac{2}{3}$ of the first and the last $\frac{1}{2}$ of the second.

22. A and B hire a pasture for \$80. A puts in 4 yoke of oxen and B puts in 40 sheep. If an ox is equal to 8 sheep, how much ought each to pay?

23. A, B and C enter into partnership with a combined capital of \$60,000. At the end of a year A's share of the profits is \$2600, B's share \$3400 and C's share \$4000. What capital did each invest?

24. If 15 men can do as much work as 21 boys, how long will 25 men take to do what 30 boys do in 14 hrs.?

25. A's rate of working is to B's as 4 to 3, and B's is to C's as 2 to 1. How long will it take C to do what A would do in 6 days?

XIV.

COMPOUND PROPORTION.

Resolve by cancelling :—

1. $6 : 4 :: 20$
   $8 : 9$

2. $7 : 12 :: 6$
   $9 : 14$

3. $\frac{5}{6} : \frac{3}{4} :: \frac{9}{10}$
   $3\frac{1}{4} : \frac{2}{5}$
   $\frac{11}{12} : 7\frac{1}{2}$

4. $\dot{9} : \dot{8} :: \dot{6}\dot{2}$
   $\dot{8} : \dot{7}$
   $\dot{7} : \dot{6}$

5. $17 : 51 :: \$13$
   $40 : 35$

6. $18\frac{3}{7} : 49 :: 3$ lbs. $5\frac{2}{3}$ oz. Troy
   $28 : 43$

7. $13 : 24 :: 52$ weeks
   $36 : 73$
   $365 : 10\frac{1}{2}$

8. $53 : 22 :: 35$ yds. 1 ft.
   $1\frac{1}{10} : 2$
   $1\frac{3}{7} : 7$

9. If 15 horses consume 360 bushels in 252 days, how many horses will be kept with 220 bushels for 154 days?

10. If 15 horses consume 360 bushels in 252 days, how many bushels will 8 horses eat in 210 days?

11. If 15 horses consume 360 bushels in 252 days, how long will 12 bushels last 7 horses?

12. Working 10 hours a day, 14 men can finish a piece of work in 12 days; how many men working 6 hours a day will be required to finish it in 35 days?

13. Working 2 hrs. a day, 180 men can build 480 yds. of wall in 8 dys.; how many men, working 10 hrs. a day, can build 450 yds. of the wall in 18 days?

14. A man can do a certain piece of work in 18 days of $9\frac{1}{2}$ hours each; in how many days of $10\frac{1}{4}$ hours each could 3 men do four times as much?

15. If \$240 gained $\$53\frac{1}{3}$ in 365 days, in what time would \$225 gain \$30 at the same rate?

16. 12 men can dig a trench 108 ft. long, $6\frac{1}{4}$ ft. deep and 3 ft. broad in 5 days; how long will 15 men take to dig one 100 ft. long, $3\frac{3}{8}$ ft. deep and 2 ft. broad?

17. The cost of carpeting a room 19 ft. by 15 ft. with carpet at 84 cents the yard is \$67.20; what will be the cost of carpeting a room 24 ft. by 20 ft. if the price of the carpet is \$1.14 a yard?

18. A lends B \$400 for 15 mos. at 4%; how long in return should B lend A \$1500 at 3%?

19. A locomotive making 162 strokes per minute travels 90 miles in 2 hours; how many strokes per minute must it make to travel 200 miles in $4\frac{1}{2}$ hours?

20. If 3 men or 5 women or 8 boys can weed 18 acres in 9 days, how long would it take 5 men, 8 women and 3 boys to weed $109\frac{1}{4}$ acres?

21. If 42 lbs. of raisins cost £1. 16s., what would 35 lbs. of a higher grade cost, 5 lbs. of the former being equal in value to 3 lbs. of the latter?

22. An officer wished to convey 80,000 lbs. of provisions in 9 days; at the end of 6 days, 10 men having been employed, 15 tons only had been carried. How many men would be required to carry the remainder within the time specified?

23. 12 women make 7 dresses in 8 days; in what time could 15 girls make 5 such dresses, the work of 2 women being equivalent to that of 3 girls?

24. The travelling expenses of 7 tourists for 5 weeks amounted to \$1505; a second party of 18 made the same tour in 6 weeks, their average weekly expenditure per man being $\frac{4}{5}$ of that of the first party. What were the total expenses of the second party?

25. Fifteen men do a piece of work in eight days; how many men could do $\frac{3}{5}$ of the work in $\frac{1}{2}$ of the time?

XV.

AVERAGES.

1. In a class of 24 boys 4 are 14 years old, 6 are $13\frac{1}{2}$, 7 are $12\frac{1}{2}$ and the rest $11\frac{1}{2}$. What is the average age?

2. A woman's income for 3 years is \$250 a year; for the next 5 years it is \$294, and for the next 4 years \$307. What was her average income for the 12 years?

3. In an exercise set to 35 pupils, 1 has 7 mistakes, 2 have 5, 4 have 3, 6 have 2, 8 have 1, and the rest none. Find their average number of mistakes.

4. If a tradesman sells on the first 5 days of the week, 243, 117, 112, 195 and 207 yards respectively, what must he sell on Saturday that the daily average may be 179 yds.?

5. In the month of April a man slept 7 hours on each of 16 nights, $6\frac{1}{2}$ hours on each of 8 nights and 5 hours on each of 5 nights. How long must he sleep the last night that his average may be $6\frac{1}{2}$ hours?

6. A candidate answers two examination papers, the first of which is valued at half as many marks again as the second. He gains 58% of the maximum marks on the first, and 43% of the maximum on the second. What percentage of the total marks does he gain?

7. To 112 gallons of spirits worth 21 francs a gallon a grocer added as much water as reduced the value to 16 francs a gallon; what quantity of water did he add?

8. A merchant has teas worth 54 cents and 44 cents a lb. respectively, which he mixes in proportion to 3 lbs. of the former to 2 lbs. of the latter, and sells the mixture at 52 cents a lb. What does he gain per cent.?

9. In a wholesale business a certain number of clerks receive \$50 a week, twice as many receive \$31.50, and eleven times as many receive \$14. The weekly pay-sheet amounts to \$1869. Find the number of clerks.

10. A man bought a herd of cattle, 136 in number, for \$3230; on the way home he lost 4, and 12 others, being unable to complete the journey, were sold for \$5 a head below cost. At how much per score must he sell the others so as to gain \$215 on the whole transaction?

XVI.

WORK AND TIME.

1. A can do a piece of work in 5 days, B in 6 days. How long will they take if they work together?

2. One pipe empties a cistern in 5 hours, another in 8 hours. In how long will the cistern be emptied if both are open?

3. One pipe fills a cistern in 4 hours, another empties it in 8 hours. If both pipes are open, in how many hours will the cistern be filled?

4. Two men together can do a piece of work in 5 hours, and one of them alone in $8\frac{1}{3}$ hours. How long would the other take to do it?

5. Two pipes can fill a cistern in $3\frac{1}{4}$ and $4\frac{1}{3}$ hours respectively, and a third empties it in $2\frac{2}{5}$ hours. In how long will the cistern be filled if all the taps are open?

6. The hot-water tap can fill a bath in 10 min., the cold-water tap in 12 min., and the waste-pipe can empty it in 8 min. If all three are opened, how long will it take to fill? And how long to empty again, if the hot water is then turned off?

7. A and B separately can do a piece of work in $4\frac{1}{2}$ hrs. and $2\frac{2}{5}$ hrs. A, B and C together can do it in $1\frac{17}{163}$ hrs. How long will C take to do it alone?

8. A and B together can do some work in 8 days, B and C together in 6 days, A and C together in $6\frac{6}{7}$ days. How long would each take by himself?

9. A and B together can do a piece of work in $8\frac{2}{11}$ days, B and C together in $9\frac{9}{19}$ days, A and C together in $8\frac{4}{7}$ days. How long would each take by himself?

XVII.

DISTANCE AND TIME.

1. A and B are 6 miles apart, and walk at the rate of $4\frac{3}{4}$ and $3\frac{1}{4}$ miles an hour respectively. How long will elapse before they come together (1) if they walk towards each other, (2) if they walk in the same direction?

2. A walking 5 miles an hour starts from Beaconsfield for Montreal (16 miles) at the same time as B walking $4\frac{1}{2}$ miles an hour starts from Montreal for Beaconsfield. How far from Montreal will they meet, and how long after the start?

3. In the preceding example, if A walked $3\frac{3}{4}$ and B 5 miles an hour, where and when would they meet?

4. A walking $5\frac{1}{4}$ miles an hour gives B walking $3\frac{3}{4}$ miles an hour one hour's start. How long will A take to catch B, and how far will he have walked?

5. At 10 A.M. a train starts from Montreal to Quebec (189 miles) at the rate of 48 miles an hour, and another at 12.30 M. from Quebec to at the rate of 44 miles an hour. How far from Montreal will they meet, and at what time?

6. A takes 9 steps while B is taking 8, but 10 of B's are equal in length to 11 of A's; which is the faster walker?

7. A train starts from a terminus at 9 A.M. travelling 25 miles an hour. An express starts at 10.30 A.M. and travels 43 miles an hour. At what time and how far from the terminus will the express overtake the slow train?

8. In a mile race A runs at the rate of 6 yards a second, and gives a start of 140 yds. to B, who runs $5\frac{1}{2}$ yds. a second. How far from the winning post will A overtake B, and by how much will he win?

9. An express starting at 3 P.M. stops first at a station $77\frac{1}{2}$ miles distant at 4.27 P.M.; the whole journey is 194 miles, and 15 per cent. of the time is expended in stoppages. At what time is the train due at the terminus?

10. An express runs $393\frac{3}{5}$ miles in 9 hours, making one stoppage of 30 minutes, three of 5 minutes each, and one of 3 min. What is its average speed when in motion?

11. A man rode a bicycle from A to B, a distance of 54 miles, at an average rate of 8 m. an hour. Another man started from A on horseback $\frac{1}{2}$ hour after the bicyclist, and arrived at B 15 min. before him. Find the ratio of their speeds.

12. A hare pursued by a greyhound was 87 yards in advance at the start; but for every $8\frac{1}{2}$ yds. which the hare ran the dog ran 10 yards. How far had the dog run when the hare was caught?

13. When will the hands of a clock be together (*a*) between 2 and 3, (*b*) between 6 and 7, (*c*) between 10 and 11?

14. When will the hands of a clock be opposite one another (*a*) between 3 and 4, (*b*) between 7 and 8, (*c*) between 9 and 10?

15. When will the hands of a clock be at right angles to one another (*a*) between 4 and 5, (*b*) between 10 and 11, (*c*) between 1 and 2.

16. Two clocks point to 2 o'clock at the same instant

on the afternoon of Christmas Day; one loses 8 sec. and the other gains 7 seconds in 24 hrs. When will one be half-an-hour before the other, and what time will each clock then show?

17. A watch which at 9.30 A.M. on Tuesday is 4 min. $8\frac{9}{32}$ sec. too fast, loses 2 min. 45 sec. daily. What time will the watch indicate at 5.15 P.M. the following Friday?

18. Two clocks, one of which gains 1 min. 12 sec. and the other loses 1 min. 28 sec. daily, are set right at 11 A.M. on 1st May; on what day and at what hour will the times indicated by them differ by 30 minutes?

XVIII.

EXCHANGE.

1. If 23·85 francs are exchanged for \$5, how many francs will be obtained for \$51.25? and how many dollars for 17·49 francs?

2. If \$4.95 are exchanged for £1 and £1 for 25·05 fr., how many francs will \$33 yield?

3. A person goes to France with £56, which he exchanges at the rate of $25\frac{1}{8}$ francs for £1. He stays 30 days, spending $37\frac{1}{2}$ fr. a day, and changes what he has at the rate of 1 fr. for $9\frac{1}{2}$d. How much English money will he have?

4. If 5 fowls are worth 3 ducks, 14 ducks worth 5 geese, and 3 geese worth 2 turkeys, what is the price of a fowl when a turkey costs \$2.10?

5. If 2 lbs. of tea were worth 3 lbs. of coffee, and 4 lbs. of coffee worth 21 lbs. of cocoa, and 7 lbs. of cocoa worth 9 lbs. of sugar, and 20 lbs. of sugar worth $4\frac{2}{5}$ lbs. of raisins, how many lbs. of raisins would be worth 24 lbs. of tea?

XIX.

MENTAL PRACTICE IN AVERAGES, PROPORTION, ETC.

1. Find the average of:—(*a*) 11, 17·25, $18\frac{1}{2}$, $19\frac{3}{4}$; (*b*) $\frac{1}{2}$, 100, 1000; (*c*) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$; (*d*) $6-3\frac{1}{2}$, $4-1\frac{1}{2}$, $3-\frac{1}{2}$.

2. What is the average attendance for a week in a school which has the following for each day: 80, 82, 81, 83, 85? If the number of pupils on the roll is 90, what is the average percentage of absence?

3. The temperature at 6 A.M. was 39° Fahr.; at noon 47·5; and at 6 P.M. 36. What was the average for the day?

4. The average of three numbers is $7\frac{1}{2}$; the largest and smallest together make up $\frac{2}{3}$ of the whole; what is the middle number?

5. If 27 out of 1000 die annually in one town, and 26 in another, what is the average death-rate per cent. per annum in the two towns?

6. At an examination there were 4 candidates at the age of 19, 3 at 20, 2 at 21, and 3 at 23. Find average age.

7. Find the missing term in the following:—(*a*) $3 : 5 :: 1\frac{1}{2} : x$; (*b*) $\frac{2}{7} : \frac{3}{14} :: \frac{2}{5} : x$; (*c*) $6 : 5 :: \cdot 3 : x$.

8. Divide:—(*a*) \$350 in the ratio of 3 : 4; (*b*) £10 in the ratio of 2 : 13; (*c*) \$75 into parts proportional to 2, 5, 8; (*d*) \$21 into parts proportional to ·25 and $\frac{1}{3}$.

9. One candle lasts 4 hrs. 20 min., another lasts 3 hrs. 15 min. What is the ratio of the first to the second?

10. A man can do a piece of work in $4\frac{2}{5}$ days. What part of it can he do in $1\frac{1}{4}$ dys.? What decimal? What per cent.?

11. A window is 6 ft. 4 in. high by 4 ft. 2 in. wide. What is the ratio of the height to the width?

12. What is the ratio of 3 quarts to 1 gal.? 6 pks. to 5 bus.? 1s. to \$1? $\frac{7}{10}$ to $\frac{3}{5}$? \$36.50 to \$18.25?

XX.

MENTAL PRACTICE IN PERCENTAGES.

1. How much per cent. is 17 of 25; 31 of $33\frac{1}{3}$; $19\frac{1}{2}$ of 20; 36 of 40; 21 of 24; $1\frac{1}{2}$ of 3; $\frac{1}{8}$ of $12\frac{1}{2}$; 18 of 63; 39 of 75; ·05 of 1; $1\frac{1}{3}$ of $5\frac{1}{3}$?

2. Express as a common fraction in lowest terms:— 2%; $4\frac{1}{2}\%$; 5%; 6%; 10%; 20%; 25%; 30%; $33\frac{1}{3}\%$; 75%; $66\frac{1}{3}\%$; $16\frac{2}{3}\%$; $12\frac{1}{2}\%$; $37\frac{1}{2}\%$; $62\frac{1}{2}\%$; $87\frac{1}{2}\%$; $9{\cdot}09\%$; $18{\cdot}18\%$.

3. Tea is bought at 4s. and sold at 3s. 4d.; what is the loss per cent.?

4. Tea is bought at 4s. and sold at 4s. 8d.; what is the gain per cent.?

5. In a school of 160 children $12\frac{1}{2}\%$ are absent; how many are present?

6. In a school of 78 pupils 26 are girls; percentage of boys?

7. In a school of 88 pupils 33 cannot write; what percentage can?

8. In a school of 250 pupils, 6 per cent. learn Latin, 8 per cent. mathematics, and 20 per cent. geography; how many in each branch?

9. The population of a town was 60,000 in 1881, and and 70,000 in 1891; required increase per cent.

10. A bankrupt's debts are \$600, his effects are \$250; how much per cent. can he pay, and how much per dollar?

11. If every shilling's worth of goods yield 2d. of gain, what is gained on £75?

12. Sold goods for \$200, by which \$25 was gained; find the gain per cent.

13. Sold at \$60 and gained \$12; find gain per cent.

14. What was lost on £25 worth of sugar bought at 5d. a lb. and sold at $4\frac{1}{2}$d. a lb.?

XXI.

COMMISSION.

(For Percentage, Interest, Discount, Present Worth, etc., see Book III., pp. 36–60.)

A Commision Merchant or **Agent** is a person who buys or sells goods for another. What he receives for his services is called his *commission.*

A **consignment** is goods sent to an agent to sell.

The **net proceeds** are the amount left after the commission and other charges have been paid.

1. A sold B's farm for $6750. He bought him a new farm for $4825. The commission for selling was 4% and for buying 2%. How much should A receive?

2. Who is the *Principal* and who the *Agent* in the above transaction? Is he a buying or selling agent? What are the net proceeds of each transaction?

3. Find the commission (*a*) at $4\frac{1}{2}$% on sales amounting to $3468, (*b*) at $5\frac{1}{2}$% on 375 barrels of apples at $2.25 a barrel, (*c*) at $6\frac{1}{4}$% on a ton of wool at $87\frac{1}{2}$ cents a lb.

4. What is the amount of the sales (*a*) when the commission at $6\frac{1}{4}$% is $150, (*b*) when the commission at $3\frac{1}{2}$% is $294, (*c*) when the commission at $1\frac{1}{2}$% is $270?

5. Find the amount of the sales (*a*) when the net proceeds are $4845 and the commission 5%, (*b*) when the net proceeds are $229.80 and the rate 3%, (*c*) when the net proceeds are $15,250, the rate of commission $1\frac{1}{4}$%, with additional charges amounting to $62.40.

6. A merchant remits to his agent a sum of money to be invested after deducting his commission. What sum will be left to be invested (*a*) when the remittance is $7098 and the commission 4%, (*b*) when the remittance is $4908

and the commission $4\frac{1}{2}\%$, (*c*) when the remittance is \$4454 and the commission $2\frac{1}{2}\%$?

7. In the above problems what represents *the base, the percentage, the rate %, the amount, the difference?*

8. Make rules or formulas for finding the *commission, the amount of sales, the sum invested.*

9. A real estate agent receives \$95 for selling a house for \$4750. What is his rate of commission?

10. What was the cost of printing 500 copies of a book which was sold at \$0.90 a copy, if the bookseller's commission and charges were 34 per cent. of the gross receipts, and the author's profit \$135.90?

11. An agent, selling goods at $2\frac{1}{2}$ per cent. commission, sent the consignor \$1267.50 as the net proceeds of 5 days' sales. What were the average daily sales?

12. An agent sold goods for me amounting to \$10,860. He charged $2\frac{1}{2}\%$ commission for selling, and $2\frac{1}{2}\%$ for guaranteeing payment, and \$37.50 for freight and storage. How many barrels of flour at \$5 a barrel can he buy with the net proceeds, if he charges 1% commission for buying?

XXII.

INSURANCE.

Insurance is security against loss.

The **premium** is the sum paid for insurance.

The **policy** is the written contract between insurer and insured.

1. What premium must be paid for insuring property (*a*) worth \$5000 at $\frac{1}{2}$ per cent., (*b*) worth \$800 at $1\frac{1}{4}$ per cent., (*c*) worth \$65,000 at $\frac{3}{4}$ per cent.?

2. What is the rate of insurance (*a*) if \$15 is paid for insuring \$1000, (*b*) if \$420 is paid for insuring \$18,000?

3. What amount of insurance can be obtained (*a*) for \$40 at 2%, (*b*) for \$157.80 at $1\frac{1}{2}\%$, (*c*) for \$187 at $2\frac{3}{4}\%$?

4. In each of the above problems point out the *base*, *rate* and *percentage*.

5. Make rules or formulas for finding the *premium*, the *rate*, the *amount of the policy*.

6. Find at sight the premium in the following cases:

Amount of Policy.	Rate.	
(*a*) \$100	10%	
(*b*) \$1200	$2\frac{1}{2}\%$	(9 examples.)
(*c*) \$2000	6%	

7. Find at sight the amount of policy in the following:

Premium paid.	Rate.	
(*a*) \$15	4%	
(*b*) \$2.50	$2\frac{1}{2}\%$	(9 examples.)
(*c*) \$10.50	$12\frac{1}{2}\%$	

8. Find at sight the rate $\%$ in the following:

Amount of Policy.	Premium paid.	
(*a*) \$50	\$5	
(*b*) \$200	\$15	(9 examples.)
(*c*) \$1000	\$25	

9. What sum must be insured at 4%, so that the owner may receive, in case goods worth \$735 are lost, the value of the goods and the premium?

10. A dealer shipped 1000 barrels of flour worth \$6.50 a barrel; for what sum must he take out a policy at $2\frac{1}{2}\%$ to cover the value of both flour and premium?

11. Find the premium for insuring 4840 bushels of wheat worth \$1.20 a bush. at $1\frac{1}{8}\%$ on $\frac{5}{8}$ of its value.

12. After 20 years' insurance, a mill worth \$48,000, and insured for $\frac{3}{4}$ of its value at $2\frac{1}{2}\%$, is destroyed by fire. Find the owner's loss, not counting interest.

XXIII.

A.—TAXES.

A **Tax** is money assessed upon the person, property or income of citizens for public purposes.

1. What is meant by *real estate, personal property, assessors?*

2. The rate of taxation on real estate in Montreal is 1% for municipal purposes and $\frac{1}{4}$% additional for school purposes. Find at sight the amount levied on properties assessed at \$1000, \$2000, \$3000, up to \$10,000 respectively.

3. When the rate of assessment is 14·8 mills on the dollar, what will be paid on a property valued at \$12,000?

4. The schools cost \$3572.50 a year, and the rateable value of real estate is \$81,657.50. What is the rate of school tax?

5. A tax of \$5900 is to be levied for repairing the roads. The assessed value of the district is \$2,242,000. What is the tax on a farm valued at \$6750?

6. What is the valuation of a piece of property that pays a tax of \$273 at the rate $3\frac{1}{4}$ mills on the dollar?

7. If a tax of \$2850 is to be raised, and the collector receives 5% commission for collecting the taxes, what sum must be levied?

B.—DUTIES OR CUSTOMS.

Duties are taxes levied by a government upon goods imported from foreign countries.

1. Define *tare, leakage, breakage.*

2. What is the duty on 182 cases indigo, each containing 150 lbs., at 75 cents a lb.?

3. What is the duty on 610 gallons of olive oil, at 25 cents a gallon, allowing 2% for leakage?

4. What is the duty at 5 cents a lb. on 400 sacks of coffee, each containing 63 lbs., the tare being 2%?

5. Find the *ad valorem* duty at $2\frac{1}{2}$% on 110 boxes of raisins, 25 lbs. in a box, invoiced at $0.12 a lb., the tare being $3\frac{1}{2}$ lbs. a box.

6. A lady brought from France 2 dozen pairs of gloves, for which she paid 6 francs a pair. The duty was $2.25 a dozen and 50% *ad valorem*. What did the gloves cost her in Canadian money, the franc being reckoned at $.193?

7. A Montreal fruit dealer received from Florida 8 boxes of oranges at $3, and 15 cases of bananas at $2.50. Each box of oranges contained 250 and each case of bananas 20 dozen on the average. If the duty is 15% and other charges $12.50, how much will be gained by selling the whole at $0.25 a doz.?

8. Find the duty at 30% *ad valorem* on an invoice of English goods amounting to £1500. 10s. 6d. (£1. = 4.866\frac{1}{2}$).

9. What is the duty on an invoice of china from Vienna, the value of which is 6429 florins, at 38%, a florin being worth $.407?

10. Required the duty and total cost of a case of French silks, value 3500 francs, duty 50% *ad valorem*, and a case of velvets, value 28,000 francs, duty 50%, other charges being 625 francs and commission $2\frac{1}{2}$%, if a franc is worth $.193.

11. Duty on a case of woollen goods from Germany invoiced at 8437 marks at 45%, a mark being worth $.238.

12. What is the invoice cost of goods upon which $360 is paid in duty, if the duty is 25% *ad valorem*?

XXIV.

STOCKS.

An incorporated company consists of several persons who are authorised by law to transact business as a single individual.

Stock is the name given to the capital of incorporated companies. The capital stock of a company is usually divided into shares of $100. The stock is said to be **at par** when a share of $100 sells for $100 in money; at a **premium,** when it sells for more than $100 in money; and at a **discount,** when it sells for less than $100 in money. The premium or discount is the difference between the quoted price and 100.

Consols are English government securities.

1. What is meant by *three per cent. stock at 85?* Is the stock at premium or discount?

2. Find (*a*) the market value of, (*b*) the annual income arising from :—

   (*a*) $620 5 per cent. stock at 85.
   (*b*) $204 $2\frac{1}{2}$ per cent. stock at 90.
   (*c*) $6000 $3\frac{3}{4}$ per cent. stock at $87\frac{5}{8}$.
   (*d*) $1550 3 per cent. stock at $75\frac{3}{8}$.
   (*e*) $1200 4 per cent. stock at $92\frac{5}{8}$.

3. Find the amount of stock obtained and the annual income derived from investing :—

   (*a*) $910 in 4 per cent. stock at 104.
   (*b*) $1560 in 5 per cent. stock at $81\frac{1}{4}$.
   (*c*) $1991.75 in 3 per cent. stock at $96\frac{3}{8}$.

4. Find the price of stock when

   (*a*) $4900 stock can be bought for $3907.75.
   (*b*) $2600 stock can be bought for $2525.25.
   (*c*) £6895 stock can be bought for £7050. 2s. 9d.

5. Find the quantity of stock held

(*a*) In the 5 per cents., if the income is \$43.75.

(*b*) In the 3 per cents., if the income is \$7.70.

(*c*) In the $5\frac{1}{4}$ per cents., if the income is \$91.

(*d*) In the 4 per cents., if the income is £115. 4s. 2d.

6. Find what sum of money must be invested to derive an income of

(*a*) \$400 from the $4\frac{1}{2}$ per cents. at $75\frac{3}{4}$.

(*b*) \$500 from the 4 per cents. at $90\frac{1}{2}$.

(*c*) \$300 from the $2\frac{1}{2}$ per cents. at $57\frac{7}{8}$.

(*d*) £72. 15s. from the $3\frac{3}{4}$ per cents. at 95.

7. Find price of stock, if there is derived an income of

(*a*) \$106.50 by investing \$2560.43$\frac{3}{4}$ in the 3 per cents.

(*b*) \$197.50 by investing \$4374.62$\frac{1}{2}$ in the 4 per cents.

(*c*) \$154 by investing \$3432 in the $3\frac{1}{2}$ per cents.

(*d*) £97. 10s. by investing £2392. 16s. 3d. in the 3 per cents.

8. Determine the rate per cent. paid by stock

(*a*) When \$3720 of stock yields an income of \$139.50.

(*b*) When \$2975 of stock yields an income of \$133$\frac{7}{8}$.

(*c*) When \$11 is obtained by investing \$264 at 81.

(*d*) When \$46.20 is obtained by investing \$924 at 105.

9. What is the rate of interest on money per cent. per annum

(*a*) When 3 per cent. stock sells at 80?

(*b*) When 3 per cent. stock sells at 84?

(*c*) When $3\frac{1}{2}$ per cent. stock sells at 75?

(*d*) When $4\frac{1}{2}$ per cent. stock sells at 135?

10. Which is the better investment:—

(*a*) The $3\frac{1}{2}$ per cents. at 77 or 4 per cents at $93\frac{1}{2}$?

(*b*) The 3 per cents. at 72 or 4 per cents. at 96?

(*c*) The 3 per cents. at $82\frac{1}{2}$ or $3\frac{1}{2}$ per cents. $93\frac{1}{2}$?

(*d*) The $3\frac{3}{4}$ per cents. at 90 or 3 per cents. at 75?

(*e*) The $4\frac{1}{2}$ per cents. at 126 or $3\frac{1}{3}$ per cents. at 90?

11. Find the change in income caused by transferring

(*a*) \$3000 from the 4 per cents. at 90 to the $3\frac{1}{2}$ per cents. at 81.

(*b*) \$7300 from the 3 per cents. at 96 to the 5 per cents at $106\frac{2}{3}$.

(*c*) \$2300 from the $3\frac{1}{2}$ per cents. at $89\frac{1}{2}$ to the $3\frac{3}{4}$ per cents. at $94\frac{3}{4}$, brokerage $\frac{1}{8}$ per cent. on each transaction.

(*d*) \$880 from the $4\frac{3}{4}$ per cents. at $106\frac{3}{8}$ to the 4 per cents. at $93\frac{3}{8}$, brokerage $\frac{1}{8}$ on each transaction.

(*e*) \$4900 from the $3\frac{1}{2}$ per cents. at $109\frac{1}{2}$ to the 3 per cents. at $91\frac{3}{4}$, brokerage $\frac{1}{8}$ on each transaction.

12. How much 4% stock must be bought to give an income of \$320?

13. If \$5125 is invested in 6% stock at $102\frac{1}{2}$, what income will be obtained?

14. If a person buys 5% stock at 120, what rate of interest does he receive on his money invested?

15. Find the sum required for an investment in 4% stock at $98\frac{1}{2}$ to produce an income of \$200 a year.

16. What must be the price of a 5% stock in order that a buyer may receive 6% on his investment?

17. When 3% Consols are quoted at 101, what sum must be invested to yield an income of £800?

18. What is the exact interest on an investment of \$5000 in $4\frac{1}{2}$ per cents. at $114\frac{1}{8}$ from Jan. 1 to March 2?

19. If a man buys stock at 17% premium, what per cent. does he receive on his investment, if the stock pays a dividend of $8\frac{1}{2}$% on its par value?

20. If 84 shares of stock, paying a 9% dividend, yield 10% on the money invested, what did the stock cost?

21. Government bonds yielding \$240 a year at 4% interest were sold at 8% premium and the proceeds invested in land at \$75 an ac. How many acres bought?

22. By selling 3 per cent. Consols at $102\frac{3}{4}$ and investing the proceeds in a railway stock which pays dividends of 7% per annum, a man finds that he can double his income; what is the price of the railway stock?

23. One company pays $5\frac{1}{2}$% on shares of \$100 each; another pays $3\frac{1}{2}$% on shares of \$10 each; if the shares of the former sell at $15\frac{1}{2}$% premium, and the shares of the latter at $22\frac{1}{2}$% discount, compare the rates of interest which the shares return to purchasers.

24. By selling out £4500 in the India 5% stock at $112\frac{1}{2}$ and investing the proceeds in Chinese 7% stock, a person finds his income increased by £168. 15s.; what is the price of the latter stock?

25. When the price of a 3 per cent. is 96, a person can obtain an annual income of \$1 more than he can if the price is 97; how much has he to invest?

26. A person has \$2950 in 3 per cent. stock at $83\frac{5}{8}$; when the stock has fallen $2\frac{1}{2}$, he transfers his capital into 5% stock at $107\frac{7}{8}$; find the alteration in his income, brokerage in each case being $\frac{1}{8}$.

27. I buy 3% stock at $89\frac{3}{8}$; after receiving one half-year's dividend, I sell the stock at $94\frac{5}{8}$, and find that I have gained \$54; what sum did I originally invest?

28. A person having 1,033,200 francs in French 3 per cents. at 74·4 transferred to English $3\frac{1}{4}$ per cents. at $98\frac{2}{5}$; find the change in his income, the rate of exchange being £1 = 25·2 francs.

29. Which is the better investment, stock at 25% discount which pays a half-yearly dividend of 4%, or money lent at 10%, interest payable annually? What % better?

30. A person sells out of the 3 per cents. at 96 and invests his money in 5 per cent. stock at par: by how much per cent. is his income increased?

XXV.

MENTAL PRACTICE IN STOCKS.

1. If the 3 per cents. are at 82, required income for $574.
2. " 3 " " 84, " " " $546.
3. " 3 " " 81, " " " $270.
4. " $3\frac{1}{2}$ " " 80, " " " $256.
5. " $3\frac{1}{2}$ " " $84\frac{1}{2}$, " " " $924.
6. " $3\frac{1}{2}$ " " 89, " " " $979.
7. " $2\frac{3}{4}$ " " 93, " " " $465.
8. " $2\frac{3}{4}$ " " 96, " " " $624.
9. " $2\frac{3}{4}$ " " 91, " " $500.50.
10. " 4 " " $87\frac{1}{2}$, " " " $350.
11. " 4 " " $85\frac{3}{4}$, " " " $686.
12. " 4 " " 89, " " " $1157.

Required the price of the stock when the

13. 3 per cents. gain $12 for $300 invested.
14. $2\frac{3}{4}$ " " $30 " $1000 "
15. 4 " " $27 " $600 "
16. $3\frac{1}{2}$ " " $42 " $1000 "
17. $3\frac{1}{2}$ " " $18 " $560 "
18. $4\frac{1}{2}$ " " $20 " $400 "

What quantity of stock can be purchased for :—

19. $300 at 75.
20. $729 at 81.
21. $656 at 82.
22. $684 at 90.
23. $760 at 91.
24. $382.50 at 85.
25. $765 at par.
26. $630 at 210.
27. $700 at 175.
28. $606 at $151\frac{1}{2}$.
29. $806 at $100\frac{3}{4}$.
30. $735 at 150.

31. I receive $3\frac{3}{4}$ p.c. interest on my money by investing in $4\frac{1}{2}$ p.c. stock. At what price did I buy ?

XXVI.

BUSINESS EXERCISES.

A.—BILLS OF ACCOUNT AND INVOICES.

A **bill of account** *is a detailed statement of merchandise sold, or of services rendered.*

An **invoice** *is a detailed statement of merchandise sold by one dealer to another.*

1. As clerk for Arthur Fitts and Co., you sell Mr. Alfred Brown, on 15th June, 1897, 3 cases of torpedoes at $2.20; 12 boxes of fire crackers at $1.62½; 3 gross pin wheels at $1.35; 5 gross sky-rockets at $3.25; 2 dozen balloons at $2.25; 45 Chinese lanterns at 9¢.

Copy and fill out the following bill of account, and receipt it for the firm:—

MONTREAL, 15th June, 1897.

MR. ALFRED BROWN.

To ARTHUR FITTS & CO., *Dr.*

2. Mr. R. W. Stuart has bought the following goods of J. R. Bradley & Co.:—

Jan. 3rd, 1897.—9½ yds. flannel at 32¢ a yd.; 26 yds. calico at 41¢ a yd.

Jan. 7th.—23 yds. muslin at 81¢; 18 yds. linen at 64¢.

Jan. 18th.—15 yds. ribbon @ 19¢; ½ doz. pairs socks @ 42¢ a pair.

Rule and make out a bill dated Feb. 1st, 1897.

Receipt the bill on 5th Feb.

3. Make out a bill for the following articles bought during March and April. Supply the names of buyer and seller; also the dates:—

23½ yds. silk @ 80¢; 1⅜ yds. lace @ $2.40; 64 yds. muslin @ 6⅞¢; 8 spools silk @ 7¢; 4 pairs stockings @ 65¢; 6 yds. linen at 87½¢; ½ doz. collars @ $2.10.

4. An upholsterer charges $3.75 per day for repairing some furniture. He supplies 6 lbs. hair @ 50¢ a lb.; 17 yds. plush @ $1.75 a yd.; 3 papers tacks at 10¢ a paper; cord, gimp, etc., 93¢. He works 4 days. Make out his bill, supplying names and dates.

5. Boston, Mass., 16th May, 1897.—Messrs. F. E. Grafton & Sons, Montreal, bought of Ginn & Co., 12 sets New Readers @ $2.25; 15 sets Kindergarten Drawing Books @ $1.75; 25 Science Readers @ $0.60; 36 Practical Arithmetic @ $0.90; 18 Brand's Physiology @ $0.50; 18 Standard Elocution @ $1.20. Less discounts of 25% and 10%. Make out the invoice.

6. Ottawa, 10th Oct., 1897.—Messrs. Bulmer & Co., Montreal, bought of Gilmour & Co., 293,500 ft. pine at $42 per M.; 132,600 ft. pine, third quality, @ $16 per M.: 425,250 ft. hemlock @ $22 per M.; 83,750 ft. basswood @ $26 per M.; 48,650 ft. elm @ $72.50 per M. Less a regular discount of 37½% and 5% additional for cash. Make out the invoice.

7. Montreal 18th July, 1897.—Messrs. H. Morgan & Sons bought of A. Chisholm & Co., 3 bbls. granulated sugar @ $7.50; 17 boxes raisins @ $1.75; 4 kegs lard @ $2.15; 15 lbs. spice at 16¢; 24 boxes oranges @ $1.40; 2 bags Java coffee @ $27.20; 2 bbls. syrup @ $18.50; 12 lbs. nutmegs @ 90¢. Less special discount 10% and 5% for cash. Cartage, $1.50. Make out the invoice.

8. Examine the following form of account with James Parker:—

MR. JAMES PARKER, *Dr.* MR. JAMES PARKER, *Cr.*

1897.				1897.			
June 3	To 10 lbs. sugar @ 10c.	1	00	June 5	By 2 days' labour..	4	00
	" 2 lbs. tea @ 55c.	1	10				
June 6	" 3 lbs. coffee @ 30c.		90				

What does "Dr." mean? What does "Cr." mean? What does the item on the Cr. side of the account mean? Did he work for you or did you work for him?

9. Rule and make out the following account with John Wallace:—June 1: He owes you $2.85. June 5: You sell him 2 qts. berries @ 12¢, peas 30¢, and 2½ lbs. steak @ 28¢. June 8: He and two of his men work for you 6 hours, each at 25¢ an hour. June 9: You sell him 2½ lbs. chops @ 28¢, potatoes 20¢. June 12: He and one man work for you 3½ hours, each at 25¢ an hour. June 15: You sell him 7 lbs. lamb @ 22¢, and berries 28¢. June 21: You sell him 1¾ lb. steak @ 28¢, and he pays you on account $3. June 24: You sell him 3 pks. potatoes @ 30¢, peas 12¢, olives 35¢, and 2¼ lbs. steak @ 28¢. June 28: You sell him 6½ lbs. beef at 24¢, and berries 30¢. How does the account stand at the end of the month?

10. Charles Harrison, gardener, owes you rent for the month of May, $25; but during the month he has done 3½ days' work for you @ $3.25 per day, and has furnished 3 rose-bushes @ 75¢, 4 grape-vines @ 50¢, 11 fuchsias @ 30¢, 25 pansies @ 10¢. How does the account stand on 1st June?

11. You employ a plumber to put in a new kitchen sink. Make out his bill for labour done and materials furnished, supplying names and dates.

12. Rule and make out the following account with Benjamin Smith:—Oct. 1: You owe him $6.24. Oct. 3: You buy of him 2 doz. apples @ 25¢, ½ lb. coffee 19¢, and 10 lbs. sugar @ 6¢. Oct. 5: You buy 5 gal. oil @ 10c, gelatine 15¢, 2 lbs. rice @ 9¢. Oct. 6: You sell him 6 bush. potatoes @ 68¢. Oct. 8: You buy 1 lb. tea @ 60¢, spice 40¢, wicks 5¢. Oct. 10: You buy 10 lbs. sugar @ 6¢, cocoa 24¢, and biscuits 30¢. Oct. 15: You buy coffee 19¢, and flour $1.40, and you pay $5 an account. Oct. 19: You buy 1½ doz. lemons @ 25¢, 2 lbs. raisins @ 13¢, biscuits 20¢, 2 lbs. brown sugar @ 7¢, and sell him 18½ lbs. butter @ 28¢. Oct. 24: You buy 5 gal. oil @ 10¢, walnuts 20¢, and 3 lbs. oatmeal @ 6¢. How does the account stand on Oct. 31st?

B.—NOTES, DRAFTS, CHEQUES.

***A note** is a written promise to pay a specified sum at a certain time.*

$730. MONTREAL, 3rd August, 1897.

Three months after date I promise to pay *Henry Webster*.....or order *seven hundred and thirty*.....dollars, at the office of the Bank of Montreal, for value received.

JOHN COX.

The above note is payable at the Bank of Montreal 3 months and 3 days after 3rd Aug., or Nov. 6th. It is negotiable, but needs Henry Webster's endorsement to make it transferable. It does not bear interest.

1. Explain the terms *negotiable* and *endorser*. What does an endorser do by his act?

2. Distinguish between *bearer* and *order*.

3. John Cox discounts the note on Sept. 15th at 9%. What will the proceeds be? (See Book III., p. 56.)

4. A note for $4000, dated 3rd July, payable in 60 days, is discounted 1st August at 12%.

Write out the note, supplying names.

Find the day of maturity, the discount and the proceeds.

5. Examine the following note:—

$250. HALIFAX, Feb. 10th, 1897.

Sixty days after date we jointly and severally promise to pay to the order of *Mr. Peter Smith, two hundred and fifty* dollars, for value received, with interest at six per cent.

MORTIMER JACK.
RICHARD SHAW.

If the words "*and severally*" were omitted, the above note would be a *joint note*, and each of the makers would be responsible for one-half of the amount. Either of the makers of the above note could be sued for the full amount. The note bears interest from its date at 6% per annum. If the note is discounted, the discount will be computed on the face of the note with the interest added.

6. James Allen borrows $337 from Richard Lee, and gives him a note at 3 months, which for his better security is signed not only by himself, but by his friend, Fred. Harris. The note is to bear interest at 7%.

Write the note supplying dates, and make it payable to order. When will it be due and what amount will be due at maturity?

7. Richard Lee (after holding the note for 18 days) has it discounted at 8%, and receives cash in return.

Endorse the note before discounting.

How much cash should Lee receive?

8. *A* **draft** *is a written order by one person on another for the payment of a specified sum.*

$100. MONTREAL, 16th Oct., 1897.

At sight.....pay to the order of *Mr. William Davidson*.....
one hundred.....dollars,
for value received, and charge to account of

To Messrs. David Law & Co., HERBERT BOND.
Toronto.

This is a sight draft, and is paid on presentation. It must be endorsed by William Davidson.

$100. MONTREAL, 16th Oct., 1897.

Thirty days after sight pay to the order of Mr. William Davidson one hundred dollars, for value received, and charge to account of

To Messrs. David Law & Co., HERBERT BOND.
Toronto.

This draft must be presented to David Law & Co. for acceptance. They accept the draft by writing across the face the word "accepted," with the date and their signature. When accepted the draft becomes a note, payable 30 days after the date of acceptance.

9. *A* **cheque** *is a draft on a bank, payable on demand.*

$150.75. MONTREAL, 2nd March, 1897.

THE BANK OF MONTREAL.

Pay to *Thomas Nicholson, Esq.*.....or order
one hundred and fifty.....$\frac{75}{100}$ dollars.

ROBERT LINTON.

This cheque must be endorsed by Nicholson before being presented for payment.

10. Draw a cheque on the Molsons Bank for $57.80, supplying names and dates.

11. MONTREAL, March 18th, 1897.—Messrs. Henry Gold & Co. buy of E. H. Dods 320 barrels of flour at $5.20, and give in part payment their cheque for $1000.

Make out Henry Gold's & Co.'s bill, and credit the amount paid. Draw the cheque.

12. MONTREAL, 3rd April, 1897.—E. H. Dods draws a draft at 30 days after sight on Henry Gold & Co. for the balance of his account. The draft is in favor of Messrs. R. Green & Sons, Toronto.

Write the draft and accept it for Henry Gold & Co., date 5th April.

13. TORONTO, 8th April, 1897.—Messrs. R. Green & Sons have the draft discounted at the Imperial Bank at 10% discount, and receive cash in return.

Endorse the draft. What are the proceeds?

14. Draw a draft, at 20 days sight, in your own favor, and have it discounted at Molsons Bank at 8%, supplying names, dates and amount. See that it is properly accepted and endorsed. Find the proceeds.

15. MONTREAL, May 10th, 1897.—Charles Wood buys of Messrs. Gillespie & Sons 10 hhds. sugar, each 1550 lbs., at 4½¢, and 20 chests tea, each 62½ lbs., at 53¢. On July 5th Wood gives his cheque on Bank of Commerse in payment. Write Gillespie's invoice and Wood's cheque.

16. QUEBEC, Aug. 19th, 1897.—Mr. S. Brush has on deposit in the Quebec Bank $193.84. He draws from his account $25.30 for himself. On the same day he deposits $422.85, and issues a cheque in favour of James Hall for $14.90. Write both cheques. How does his account stand?

17. THREE RIVERS, Apr. 24th, 1898.—J. Ross borrows $425 from H. Bird, and gives his note at 90 days, drawing interest at 8%, in payment. At maturity Ross pays $132 to Bird, and gives a new note at 2 mos., drawing interest at 10%, for the balance. Ross pays the second note by a cheque on the Quebec Bank. (*a*) Write the first note. (*b*) Write the second note. (*c*) Write the cheque.

XXVII.

WEIGHTS AND MEASURES.

(See tables on p. 76, with notes.)

1. How many loads of coal, each 14 cwt. 96 lbs., are contained in 5 trucks, each weighing 10 tons 8 cwt.? *(Long ton.)*

2. A dealer bought 600 tons of coal at $5.25 a long ton, paid 75¢ a ton for freight, and sold it for $5.75 a short ton. Find his profit.

3. Sea water contains $2\frac{1}{2}\%$ of salt; what weight of water would be required to yield half a ton of salt?

4. Find the weight of a million cent-pieces, each weighing a quarter of an ounce.

5. Twelve tons (gross) of tobacco were sent to an army of 21,000 soldiers; how many lbs. should each receive?

6 A silk-worm produces 28 grains Troy of silk; how many must be kept to produce 112 lbs. Avoir.?

7. Express in Troy weight the difference between 140 lbs. and 2120 oz. Avoir.

8. Reduce the sum of 306 lbs., 306 oz., 306 dwts. and 306 grs. to Avoirdupois weight.

9. Find the difference in grains between the weight of a pound of feathers and a pound of gold.

How many lbs. Troy are there in a long ton of gold?

10. Express in lbs. Avoir. the weight of a nugget of gold weighing $18\frac{2}{3}$ lbs. Troy.

11. A watch gains 13 seconds each hour; what will it gain in a fortnight?

12. How long would it take to count ten millions, twenty thousand, three hundred, at the rate of one hundred and fifty per minute?

13. A workman goes to work at 6.30 A.M. each morning and leaves at 8.15 P.M. What does he earn in 4 days if he is paid 18 cents per hour till 6 P.M., and after that time 24 cents per hour, but loses $1\frac{1}{2}$ hours of working time each day for breakfast and dinner?

14. A train travelling 45 miles an hour continues its journey for $2\frac{1}{2}$ hrs., stopping twice for 7 min. 30 sec. each time. What distance is traversed?

15. A train leaving at 8.05 A.M. arrives at its destination at 2.45 P.M., travelling on an average half a mile per minute. What is the distance traversed?

16. If sound travels at the rate of 1090 ft. a sec., and if a gun is discharged $5\frac{1}{4}$ mi. away, what time will elapse after seeing the flash before hearing the sound?

17. If light travels 186,000 miles a sec., how long would it take to pass from the sun to the earth, a distance of 92,000,000 miles? How long to go round the earth, a distance of 24,899 miles?

18. From a rod a yard long, pieces each ·057 of an inch long are cut off; how many such pieces can be cut off, and what will be the length of the remaining piece?

19. How many ropes, each 24 yds. 1 ft. 6 in. will reach to a depth of 294 fathoms?

20. How often is a chain contained in a mile and a half? What is the length of a chain in yards? In rods?

21. How many inches long is a link?

22. The distance between two places is found to be 1000 chains. Express the distance in miles, etc.

23. The four sides of a garden are 25 chains 19 links, 15 chains 8 links, 24 chains 98 links, and 16 chains 4 links. How many yards around the garden?

24. Find in acres and sq. chains the area of a rectangular field 1769 links long and 1200 links broad.

25. A path is 15 links wide and 88 chains 40 links long. Find the area of the path in sq. rods.

26. From a field of 6 acres, a rectangular piece 4·25 chains long and 2·4 chains wide is fenced off. How much is left?

27. One field contains 4 sq. chains and another is 4 chains square. How many acres in both together?

28. Take ·432 of an acre from $6\frac{1}{4}$ sq. chains, expressing the result in square yards and a decimal.

29. A school-room having in it 45 pupils is 26 ft. wide and 10 ft. 6 in. high. How long must it be to give 250 cub. ft. of space to each pupil?

30. How many loads of loam, each a cubic yard, will it take to cover a quarter of an acre of land 2 in. thick?

31. Find in gallons the measure to four decimal places of a cub. ft. of water, and its weight in ounces Avoir.

32. Find the weight of water in a cistern 10 ft. long 4 ft. wide, the water standing 18 inches deep.

33. What weight of water does the Suez Canal contain, if it be 100 miles long, 100 ft. wide and 25 ft. deep?

34. Find the weight of air in a room 24 ft. long, 20 ft. wide, $10\frac{1}{2}$ ft. high, if 100 cub. in. of air weighs 31 grains.

35. Find the value, at $\frac{1}{4}$d a lb., of the ice taken from a pond half an acre in extent, if the ice is 10 in. thick and one cubic foot weighs $58\frac{1}{8}$ lbs.

36. A floating body displaces its own weight of water. How many cubic feet of water will be displaced by a ship and cargo weighing 16,000 tons?

37. A ship sails 2° 31′ one day, 2° 35′ the next, 2° 33′ the next, and 2° 20′ on the fourth. What distance is traversed, reckoning 60 miles to a degree in that latitude?

38. A ship sails 195, 192, 187, 245, 243, 203 and 226 knots (2000 yds.) in a week. Find the distance in miles.

XXVIII.

METRIC SYSTEM.

(See Book III., pp. 13–23.)

LENGTH.

Mm.	Km.	Hm.	Dm.	m.
1	= 10	= 100	= 1000	= 10,000

m.	dm.	cm.	mm.
1	= 10	= 100	= 1000

Construct after the above model the table of capacity, the unit of measure being the *litre*.

Construct the table of weight, the unit being the gram.

The theory of this system is that a metre is the 10,000,000th of a quadrant of the earth through Paris; the litre is a cubic decimetre in volume; the gram is the weight of $\frac{1}{1000}$ of a litre filled with water at 4° C.; and the franc weighs 5 grams. Hence

1 litre=1 cubic decimetre=1 kilogram. (Book III., p. 22.)

1. A wagon is loaded with 5 boxes weighing respectively 102·345 kilos, 37·93 kilos, 342·61 kilos, 9·849 kilos and 125·6 kilos; what is the weight of the whole load?

2. A person paid 24 francs for 15 m. of cloth; how much will he pay for 75 cm.?

3. How many litres of vinegar can be put into 500 bottles, if 25 of them will hold 625 cl.?

4. Make out the following bill:—6 doz. dinner plates at 55 centimes each, 84 cheese plates at 4·45 fr. a doz., 3 dishes at 1·25 fr. each, 5 doz. glasses at ·65 fr. each.

5. Two bicyclists start from opposite ends of the same journey, which is 192·5 Km. long, and meet in 7 hr. 42 min. One rides 3·5 Km. an hour faster than the other; what is the speed of the faster?

6. Two trains start from Paris with an interval of 2 hrs. 21 min. The first runs 48·24 Km. and the second 54·27 Km. per hour. At what distance from Paris will the second overtake the first?

7. A lawn 27 m. long and 15 m. wide has a walk round it 2 m. wide; what will it cost to cover the walk with gravel at ·50 fr. a sq. m.?

8. If a pile of wood is 2·8 m. high and 4 m. deep, how long must it be to contain 112 steres?

9. Find the weight of water in a cistern 4 m. long, 3 m. wide and 2 m. deep.

10. How many Hl. of water will a tank hold which is 8·80 m. long, 6 m. wide and 4.40 m. deep?

11. A bin 4·5 m. long, 2 m. wide and 3 m. deep will hold how many litres of grain?

12. Each edge of a cube of brass is 5 cm. long; find its weight assuming brass to be 8 times as heavy as water.

13. Find the weight of oil in a tank 5 m. × 4 m. × 3 m., the weight of oil being 92% of the weight of water.

14. What is the weight of a bar of iron 4·60 m. × 7 cm. × 1·80 cm., specific gravity being 7·8?

15. How many franc pieces will weigh as much as a cubic foot of water?

16. A swimming-bath contained 52·70 Hl. of water: water was then allowed to run for $2\frac{1}{2}$ hrs. from a tap which supplied it at the rate of 32 l. per min. Find the quantity and weight of water then in the bath.

17. A family burned 600 kilos of coal in 40 days. If a Hl. of coal weighs 10 kilos, and 28 Hl. cost 56 fr., what was the cost of a day's fuel?

18. A cubical vessel 40 cm. on an edge is full of water. If 14 l. are drawn off and replaced by a liquid whose weight is $\frac{7}{8}$ that of water, what is the weight of the mixture?

Metric Equivalents.

1 metre	= 39·37 inches.	1 litre	= 1·76 pints.
1 kilometre	= ·6214 mile.	1 hectolitre	= 22·01 gal.
1 sq. metre	= 1·196 sq. yds.	1 gram	= 15·432 grains.
1 hectare	= 2·471 acres.	1 kilo	= 2·2046 lbs. Av.
1 cu. metre	= 1·308 cu. yds.	1 metric ton	= 1·1023 tons.

Approximate Equivalents.

Metre	= 1·1 yds.	Cu. metre	= 1·3 cu. yds.
Kilometre	= $\frac{5}{8}$ mi.	Litre	= $1\frac{3}{4}$ pints.
Sq. metre	= $1\frac{1}{5}$ sq. yds.	Hectolitre	= 22 gal.
Hectare	= $2\frac{1}{2}$ acres.	Gram	= $15\frac{1}{2}$ grains.
Cu. centimetre	= $\frac{1}{16}$ cu. inch.	Kilogram	= $2\frac{1}{5}$ lbs. Av.

19. Express a metre in yards; an inch in cm.; a yard in metres; a mile in Km. (*3 places of decimals.*)

20. Express a sq. metre in sq. inches; a sq. inch in sq. cm.; a sq. yd. in sq. m.; an acre in hectares.

21. Express a cu. cm. in cu. in.; a stere in cords; a cu. inch in cu. cm.. a cu. yd. in cu. m.

22. Express a pint in litres; a gal. in Hl.; a bu. in l.

23. Express a mg. in grains; a grain in grams; an oz. Av. in grams; an oz. Troy in grams; a lb. Av. in kilos.

24. How many miles and rods are there in 30·675 Km.?

25. Change 3 cu. yds. 4 cu. ft. to cu. m.

26. Express 10 cu. ft. 156 cu. in. in litres.

27. Express 1800 gal. in cu. m.

28. Change 1525 cg. to oz. dwt. grs.

29. Change £12·5 dl. to cu. ft. and cu. in.

30. Find as the decimal of an inch the difference between 12 inches and 30 cm.

31. If a cubic foot of glass weigh 156 lbs., find in lbs. and also in kilograms the weight of a pane of glass 24 inches long, 20 inches wide and $\frac{1}{8}$ inch thick.

32. A train goes at an average rate of 5 Km. in 6 minutes. How long will it take to go from Montreal to Ottawa, a distance of 120 miles?

33. Find the weight in kilograms of a rectangular block of gold 8 inches long, 3 inches wide and 2 inches thick, gold being $19\frac{1}{4}$ times as heavy as water.

34. A metre being 39·371 inches, show that a kilometre is ·621385732 of a mile.

35. Express a millimetre in in. and a kilometre in ft.

36. A kilogram is equal to 2·2046 lbs. Av., and a cubic inch of water weighs 252·9 grains. Hence find the number of cubic inches in a litre.

37. A litre of air weighs 1·3 grams; how many grains will a cubic foot of air weigh?

38. In 25 kilograms how many lbs. Troy?

39. A vessel full of water weighs 5·25 Kg.; the weight of the vessel when empty is 250 g. How many litres will the vessel hold?

40. A piece of iron weighing 100 lbs. is made into a bar 5 cm. wide and 2·5 cm. thick. What is its length if the specific gravity of the iron is 7·5?

41. A bottle empty weighs 10,000 grains; full of oil it weighs 1075 grams. What part of a litre will the bottle hold if the specific gravity of the oil is ·9?

42. Mention the standard units in the metric system, and explain how each of the other standards is derived from that of length.

43. How was the standard of length determined?

44. Express the metric standards in British denominations, correct to 3 places of decimals.

45. Why is the system called Metric?

46. What is its great superiority over every other system of weights and measures?

XXIX.

POWERS AND ROOTS.

A.—Squares and Cubes.

Find the square of :—

1. 41. **2.** 34. **3.** 925. **4.** $6\frac{1}{4}$. **5.** $24\frac{1}{3}$. **6.** 50·24.

Find the cube of :—

7. 31. **8.** 1·13. **9.** 10·05. **10.** $\frac{18}{19}$. **11.** $\frac{119}{598}$. **12.** 31·5.

Find the value of :—

13. 51^6. **14.** $(2\frac{1}{7})^4$. **15.** 17^9. **16.** $(\frac{5}{6})^5$. **17.** $(1\frac{7}{11})^6$.

18. $23^2+15^2+3^3$. **19.** $1{\cdot}031^3-1{\cdot}03^3$. **20.** $372{\cdot}5^2-36^3$.

21. $38^2+17^3-18^3$. **22.** $39^2\times48^3$.

23. $(397^2+397^3)\div39700$. **24.** $(15^3-1{\cdot}31^2)\div15$.

25. $1{\cdot}03\ (4{\cdot}07+3{\cdot}16)^3$. **26.** $3000\ (1{\cdot}1\times{\cdot}031)^3$.

B.—Square Roots.

Extract the square root of :—

1. 576. **2.** 1681. **3.** 1156. **4.** 4761.

5. 18769. **6.** 183184. **7.** 10404. **8.** 266256.

9. 1·592644. **10.** 234·09. **11.** ·00053361. **12.** 1073·741824.

13. $42\frac{1}{4}$. **14.** $39\frac{1}{16}$. **15.** $592\frac{1}{9}$. **16.** $113\frac{8}{36}$.

17. $\frac{144}{576}$. **18.** $\frac{256}{1296}$. **19.** $\frac{4096}{46656}$. **20.** $\frac{163844}{653376}$.

Find to *three* decimal places the value of :—

21. $\sqrt{2}$. **22.** $\sqrt{6}$. **23.** $\sqrt{796}$. **24.** $\sqrt{81{\cdot}4}$.

25. $\sqrt{4\frac{3}{8}}$. **26.** $\sqrt{8\frac{1}{3}}$. **27.** $\sqrt{\frac{4}{5}}$. **28.** $\sqrt{{\cdot}00099}$.

29. $\sqrt{328}+\sqrt{7}+\sqrt{5}$. **30.** $\sqrt{1142{\cdot}44}+\sqrt{11{\cdot}6281}$.

31. $5\sqrt{3}+3\sqrt{5}$. **32.** $\sqrt{{\cdot}97}\div\sqrt{{\cdot}90}$.

Find a mean proportional, correct to 3 places, to :—

33. 25 and 9. **34.** 8 and 18. **35.** 28 and 63. **36.** 7 and 13.

37. $\frac{1}{4}$ and $\frac{1}{9}$. **38.** ·25 and ·81. **39.** $\frac{1}{12}$ and $\frac{1}{75}$. **40.** ·12 and ·48.

41. What is the side of a square whose area contains 106,929 sq. yards?

42. A regiment consists of 15,876 foot-soldiers; how many must be placed in rank and file to form a solid sq.?

43. A square farm contains 36·1 acres; how many rods square is it?

44. Find in rods the length of a side of a square piece of land containing 262 ac. 105 sq. rods.

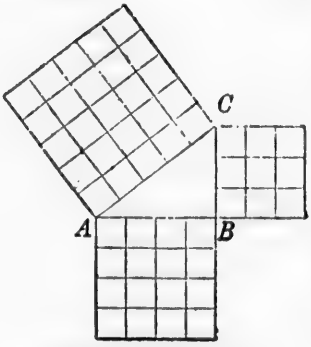
45. A rectangle is 81 yds. long by 64 yds. wide. Find the side of a square that has the same area.

46. A rectangular field containing 80 acres is twice as long as it is broad. Find its length and breadth.

$$\text{Hypotenuse} = \sqrt{\text{Base}^2 + \text{Perpendicular}^2}.$$

$$\text{Perpendicular} = \sqrt{\text{Hypotenuse}^2 - \text{Base}^2}.$$

$$\text{Base} = \sqrt{\text{Hypotenuse}^2 - \text{Perpendicular}^2}.$$



$AC^2 = AB^2 + BC^2$

$5^2 = 4^2 + 3^2$

47. If the sides of a right-angled triangle be 2 in. and 3 in. in length, find the hypotenuse correct to $\frac{1}{100}$ part of an inch.

48. If the hypotenuse be 8 ft. and base 4 ft., find the perpendicular correct to $\frac{1}{1000}$ of a foot.

49. Find the height of a window reached by a ladder 25 ft. long, whose foot rests 15 ft. from the house.

50. The height of a tree standing on a riverside is 100 ft., and a line stretched from its top to the opposite bank is 144 ft.; find the width of the river.

51. A tree was broken 35 feet from its root, and struck the ground 21 feet from its base. Find the height of the tree.

C.—Cube Roots.

Extract the cube root of :—

1. 2197.	**5.** 39651821.	**9.** 1·225043.
2. 12167.	**6.** 7077888.	**10.** 27270·901.
3. 19683.	**7.** 134217728.	**11.** ·000300763.
4. 493039.	**8.** 228099131.	**12.** 233·744896.

13. $\frac{27}{64}$. **14.** $15\frac{5}{8}$. **15.** $37\frac{1}{27}$. **16.** $13\frac{103}{125}$. **17.** $5164\frac{309}{343}$.

Find, correct to 3 places of decimals, the value of :—

18. $\sqrt[3]{73}$. **19.** $\sqrt[3]{108}$. **20.** $\sqrt[3]{\cdot 112}$. **21.** $\sqrt[3]{\cdot 003}$.

22. $\sqrt[3]{\cdot 000714} + \sqrt[3]{32}$. **23.** $\sqrt{1\cdot 44} + \sqrt[3]{30\frac{1}{4}} - \sqrt[3]{1607\frac{67}{343}} + \sqrt[3]{244\cdot 140625}$.

24. Find the cube root of the fourth power of 112.

25. What is the length of the side of a cube which contains 9 c. yds. 11 ft. 64 in. ?

26. Find the content of a cube, if the diagonal of one of its faces is $12\sqrt{2}$ inches.

27. The areas of similar surfaces are to each other as the squares of their like dimensions.

(*a*) If a pipe one inch in diameter will fill a cistern in 60 minutes, how long will a pipe 2 in. in diameter take?

(*b*) If one side of a triangle containing 36 sq. yds. is 8 yds., what is the length of a corresponding side of a similar triangle which contains 81 sq. yds. ?

28. The contents of similar solids are to each other as the cubes of their like dimensions.

(*a*) If a globe 4 in. in diameter weighs 32 lbs., what is the weight of a globe 5 in. in diameter ?

(*b*) If a sphere 3 in. in diameter weighs 4 lbs., what is the diameter of a sphere that weighs 32 lbs. ?

MENSURATION.

XXX. PLANE FIGURES.

A.—Parallelograms.

Area = length × perpendicular height. (Book III., p. 27.)

Area = base × altitude.

If h stands for altitude and b for base,

(1) area $= h \times b$, (2) $h = \frac{\text{area}}{b}$, (3) $b = \frac{\text{area}}{h}$.

If base and altitude are equal, $h = b$, then

area $= h^2$, $h = \sqrt{\text{area}}$; or area $= b^2$, $b = \sqrt{\text{area}}$.

1. The area of a square is $544\frac{4}{9}$ square yards. What is the length of its side?

2. The area of a rectangular field is 14 acres 91 sq. rds.; what is its breadth, its length being 1575 links?

3. What is the perpendicular breadth of a rhomboid $1\frac{1}{2}$ acres in area and 25,000 links in length?

4. The paving of a square court cost £1. 7s., at the rate of 1s. 4d. per square yard. How long was it?

5. 900 paving stones each 2 ft. by 1 ft. 3 in., are required for a pathway 100 yards in length. What is the width of the pathway?

6. What will be the cost of painting the walls and ceiling of a room 17 ft. long, 13 ft. 3 in. broad and 11 ft. high, containing two windows, each 6 ft. by 4 ft. 3 in., and two doors, each 8 ft. by 4 ft., at the rate of $1.08 sq. yd.?

7. Two pathways, 5 ft. wide, at right angles to one another, and parallel to the side, run across a rectangular courtyard 79 ft. by 63 ft. Find the cost of paving them at 99 cents per sq. yd. (*Draw diagram.*)

8. How many links long is a square field containing 9 ac. 81 sq. rods?

B.—Triangles.

1. *Area* $= \frac{1}{2}$ (*base* $\times$ *altitude.*) (Book III., p. 28.)

(1) area $= \frac{h \times b}{2}$, (2) $h = \frac{\text{area} \times 2}{b}$, (3) $b = \frac{\text{area} \times 2}{h}$.

2. *Area in terms of the sides* $= \sqrt{s\,(s-a)\,(s-b)\,(s-c)}$ where s is half the sum of the three sides.

Rule.—*From half the sum of the three sides subtract each side separately; multiply the half sum and the three remainders together, and extract the square root of the product.*

(For hypotenuse, base and perpendicular of right-angled triangles, see p. 54.)

1. Find the areas of the following triangles:—
   - (*a*) Base, 150 ft.; altitude, 42 ft.
   - (*b*) Sides, 12, 15 and 18 yards.
   - (*c*) Sides, 1200, 1450 and 1500 links. (Ans. in ac.)
   - (*d*) Sides, 4·5, 6·2 and 7·8 inches.
   - (*e*) Perimeter, 120 ft.; sides proportional to 5, 9, 10.
   - (*f*) Perimeter, 27 yds.; triangle, equilateral.

2. Base, 128 ft.; area, $298\frac{2}{3}$ sq. yds. Find altitude.

3. Area, 144 acres; altitude, 60 rods. Find base.

4. A board 16 feet long is 22 inches wide at one end and tapers to a point; what is its value at $4\frac{1}{2}$ cents a sq. foot?

5. Find the area of a square field whose diagonal is 380 yards.

6. A square field contains 35 acres; find its diagonal.

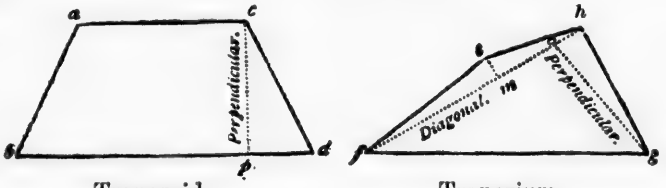
7. The area of a triangle is 6 ac. 88 sq. rods, and a perpendicular from one angle on the base measures 524 links. Find the length of the base in chains.

8. The sides of a triangular field are 300, 400 and 500 yards; if a belt 50 yds. wide is cut off the field, find the sides of the interior triangle and the area of the belt.

C.—Trapezoid and Trapezium.

A **trapezoid** is a four-sided figure having two of its opposite sides parallel.

A **trapezium** is a four-sided figure having no parallel sides.



Trapezoid. Trapezium.

1. *The area of a trapezoid is equal to half the sum of its parallel bases multiplied by its altitude.*

If b and b^1 stand for the parallel bases,

$$\text{area} = \tfrac{1}{2}\,(b + b^1) \times h.$$

2. *The area of a trapezium is equal to the diagonal multiplied by half the sum of the two perpendiculars falling upon it from the opposite angles.*

If h and h^1 stand for the perpendiculars, and d for the diagonal, $\text{area} = \tfrac{1}{2}\,(h + h^1) \times d.$

3. Prove the two rules given above by showing that both are equivalent to the following:—

The area of a trapezoid or trapezium is equal to the sum of the areas of the two triangles into which the figure may be subdivided.

4. A field is in the form of a trapezoid; its parallel sides are 10 chains 30 links and 7 chains 70 links in length, and the distance between them is 7 chains 50 links. Find the acreage.

5. Find the area of an irregular piece of ground, the diagonal of which is 320 yards and the perpendiculars 35·5 yds. and $42\frac{1}{2}$ yds.

6. Find the acreage of a trapezoid whose parallel sides are 1964 and 1250 links respectively, and whose altitude is 250 links.

7. Find the acreage of a field ABCD, right-angled at B, if AB = 525 links, BC = 440 links, CD = 875 links, and DA = 260 links.

8. In a trapezium EFGH, EF = 586, FG = 1068, GH = 766, HE = 964, and EG = 1468. Find the area.

9. A four-sided field has two sides parallel and the other two sides equal to one another; if the parallel sides are 370 and 250 links long, and each of the equal sides 100 links long, find the area.

10. The opposite sides of a quadrilateral are parallel, and the distance between them is 7 chains 50 links; if the area is 6·75 acres and the length of one of the parallel sides is 10 chains 30 sides, find the length of the other.

D.—Circle.

1. *The circumference of a circle = diameter* × $3\frac{1}{7}$ *or* 3·1416. (Book III., p. 30.)

The letter π stands for the ratio of the circumference to the diameter. Hence π * = $\frac{22}{7}$ or 3·1416.

If r stands for radius, $2r$ will stand for diameter, then

$$(1)\ \text{circumference} = 2\pi r,\quad (2)\ 2r = \frac{\text{circumference}}{\pi}.$$

2. *Area of circle* = $\frac{1}{2}$ (*circumference* × *radius*). (Book III., p. 30.)

$$= \tfrac{1}{2}(2\pi r \times r) = \frac{2\pi r^2}{2} = \pi r^2.$$

Rule.—*The area of a circle is found* (*a*) *by multiplying the circumference by half the radius, or* (*b*) *by multiplying the square of the radius by* $\frac{22}{7}$ *or* 3·1416.

3. Find the area of the circle whose radius is (*a*) 40 feet, (*b*) 1760 yards, (*c*) 5 ft. 9 in.

* In the exercises of this book $\pi = \frac{22}{7}$, unless stated otherwise.

4. Find the area of a circle whose circumference is (*a*) 200 yards, (*b*) 10 feet.

5. Find the radius of the circle whose area is (*a*) 50 sq. ft., (*b*) half an acre.

6. The radius of a circle is 10 ft., and a square is inscribed within it; find the difference between the area of the circle and that of the square.

7. How many times will a roller, whose breadth is 3 ft. 6 in. and diameter 2 ft. 6 in., have to revolve to roll a cinder-path a quarter of a mile long and 7 ft. broad?

8. A circular field contains 3 acres, and a walk round it contains $\frac{1}{2}$ acre; what is the diameter of the field, of the field and the walk, and the breadth of the walk?

9. The circumference of a circle is one mile; find its area in acres.

10. Find the circumference of a circle whose area is equal to that of a square the side of which is 320 yds. long.

11. Find the number of sq. rods in a roadway 5 yds. wide round a circular pond 120 yds. in diameter.

12. A circle, an equilateral triangle and a square have the same perimeter, 120 feet. Find their areas.

13. The diameters of the wheels of a bicycle are 52 and 15 inches respectively; determine how many more revolutions the small wheel would make than the large wheel in a distance of 13 miles.

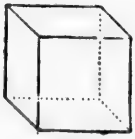
14. The area of a square is 9·8 acres; find in yards the circumference of the circumscribing circle.

15. The radius of the inner boundary of a circular ring is 14 inches; the area of the ring is 100 sq. inches. Find the radius of the outer boundary.

16. An ass is tethered in the midst of a field so that it can feed on an acre of ground; what is the length of the tether?

XXXI. SURFACES AND VOLUMES OF SOLIDS.

A.—CUBE, PRISM, CYLINDER.



Cube.

Prism.

Cylinder.

General Rules.—(A) *The area of the lateral surface of a cube, prism or cylinder is the product of the perimeter of the figure by its altitude.*

(B) *The area of the total surface of a cube, prism or cylinder is the sum of the areas of its lateral surface and its two bases.*

(C) *The volume of a cube, prism or cylinder is the product of the area of its base by its altitude.* (See Book III., p. 31.)

From the above rules we derive the following:—

1. **Cube.**—If x stands for the length of its side,
   Lateral surface $= 4x^2$ (1), *Total surface* $= 6x^2$ (2).
   Volume $= x^3$ (3).

2. **Prism.**—If p stands for perimeter of base,
   Lateral Surface $= p \times h$ (1).
   Total Surface $= (p \times h) +$ *areas of two bases* (2).
   Volume $=$ *area of base* $\times h$ (3).

3. **Cylinder.**
   Convex surface $=$ *circumference of base* $\times$ *altitude* $= 2\pi r \times h$ (1).
   Total surface $= (2\pi r \times h) +$ *areas of two bases* $= 2\pi r \times h + 2\pi r^2$ (2).
   Volume $=$ *area of base* $\times$ *altitude* $= \pi r^2 \times h$ (3).

4. Find the total surface of a cube whose edge is (*a*) 15 in., (*b*) 5 ft. 10 in., (*c*) 6 ft. 3 in., (*d*) 5 ft. 1 in., (*e*) 10 ft. 3 in.

5. Find the total surface of a parallelopipedon which measures (*a*) 10 in., 15 in., 18 in.; (*b*) 6 ft. 1 in., 2 ft. 10 in., 1 ft. 8 in., (*c*) 6 ft. 3 in., 5 ft. 4 in., 4 ft. 9 in.

6. Find the total surface of a triangular prism, the sides of whose base and height are (*a*) 2 in., 3 in., 3 in. and 1 ft.; (*b*) 2 ft. 3 in., 1 ft. 5 in., 1 ft. 5 in. and $1\frac{1}{2}$ ft.

7. Find the total surface of a cylinder whose radius and height are (*a*) 2 in. and 1 ft. 4 in.; (*b*) 1 ft. 6 in. and 9 in.; (*c*) 9 in. and 1 ft. 6 in.

8. Find the lateral surface of a hexagonal prism, each edge of base being 2 ft. 3 in. and height 5 ft.

9. If 30 cubic inches of gunpowder weigh a lb., what weight of gunpowder will be required to fill a cylinder whose internal diameter is 8 in. and length $2\frac{1}{2}$ ft.?

10. Find the length of the edge of a cubical block of stone containing 46 cub. yds. 513 cub. in., and the number of sq. inches in its entire surface.

11. Water is poured into a cylindrical reservoir 20 ft. in diameter at the rate of 400 gallons a minute. Find the rate per minute at which the water rises in the reservoir.

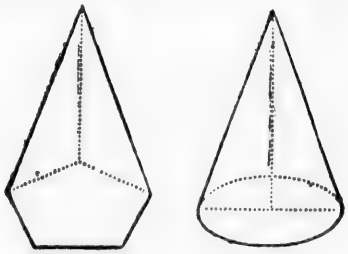
12. The length of a trough is 15 ft. and its ends are equilateral triangles; it contains 173,200 cubic feet of water. Required the depth.

13. Find the volume of a prism on a triangular base, the sides of the base being 51, 40, 13 in., and the height 58 in.

14. The surface of a cube contains 337 sq. ft. 72 sq. in.; find its volume.

15. The external measurements of the sides of a box are 3, 2·2, 1·52 ft.; find its volume. Find also the cubical space inside the box (closed by a lid), the thickness of the wood being $\frac{1}{10}$ ft.

B.—Regular Pyramid and Right Circular Cone.



Pyramid. Cone.

***A* pyramid** *is a solid, the base of which is any plane figure, and its lateral faces are triangles meeting in a point called the vertex of the pyramid.*

Pyramids, like prisms, are named triangular, square, etc., as their bases are triangles, squares, etc.

***A* cone** *is a round pyramid with a circle for its base.*

General rules.—(*a*) *The area of the lateral surface of a pyramid or cone is half the product of the perimeter of the base by the slant height.*

(*b*) *The area of the total surface of a pyramid or cone is the sum of the areas of the lateral surface and of the base.*

(*c*) *The volume of a pyramid or cone is one-third the area of the base multiplied by the altitude.*

From the above we get the following formulæ:—

1. **Pyramid.**

Lateral surface $= \frac{1}{2}$ (*perimeter* $\times$ *slant height*).

Total surface = *sum of triangular faces* + *area of base.*

Volume $= \frac{1}{3}$ (*area of base*) $\times$ *altitude.*

2. **Cone.**—If r = radius of base,

Lateral surface $= \frac{1}{2}$ (*circumference of base*) $\times$ *slant height* $= \pi r \times$ *slant height.*

Total surface $= (\pi r \times$ *slant height*$) + \pi r^2$.

Volume $= \frac{1}{3}$ (*area of base*) $\times$ *altitude* $= \frac{1}{3} \pi r^2 \times h$.

3. Calculate the entire surface of a square pyramid whose slant height is 18 inches, the area of its base being 144 sq. in.

4. Calculate the entire surface of a triangular pyramid whose three lateral faces and the base are equilateral triangles, each side measuring 2 inches.

5. Draw the developed convex surface of a cone, the diameter of whose base is 4 inches, and whose slant height is 6 inches. Find this surface.

6. How many square inches of paper would be required to cover the side and base of a cone 6 inches in diameter at the base, and having a slant height of 10 inches?

7. Find the slant height of a cone whose altitude is 12 inches, the diameter of its base being 10 inches. What is its convex surface?

How is a right circular cone generated?

NOTE.—The altitude of a cone or pyramid, the slant height and the perpendicular from the centre of the base to the side, form a right-angled triangle.

8. Required the volume of a square pyramid 15 ft. 9 in. high, the side of its base being 2 ft. 6 in.

9. What is the solidity and surface of a cone 5 feet in diameter and 12 feet high? ($\pi = 3{\cdot}1416$.)

10. A triangular pyramid has each side of the base 5 ft.; the height is 10 ft.; find its solidity and surface. ($\pi = 3{\cdot}1416$.)

11. Find the volume of a cone whose altitude is 18 metres and diameter of base 6 metres.

12. A conical tent whose slant height is 12 ft. requires 132 sq. ft. of canvas to cover it. Find how many feet of ground the tent covers.

13. It is required to cover a piece of ground 80 feet square with a pyramidal tent 30 feet in perpendicular

height. Find the cost of the requisite quantity of canvas at $4\frac{1}{2}$d. per square yard.

14. How many square yards of canvas will be necessary to form a conical tent whose perpendicular height is 10 ft. and diameter on the ground 21 ft.?

15. Find the slant height of a cone whose volume is $19\frac{1}{4}$ cub. ft. and diameter at base $3\frac{1}{2}$ ft.

16. Find the volume of a cone, the height of which is 15 ft. and circumference of base 14 ft.

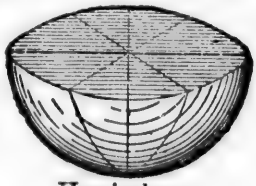
17. The diagonal of the base of a square pyramid and the diameter of the base of a cone are each 16 ft.; their altitudes are equal, but the volume of the cone exceeds that of the pyramid by 281 cub. ft. Find the altitudes.

18. The Great Pyramid of Egypt stands on a square base, each side of which is 764 feet, and its height is 480 ft. Find its volume in cubic feet.

C.—The Sphere.

Sphere.

***A* sphere** *or* **globe** *is a solid bounded by a curved surface, all points of which are equally distant from a point within called the centre.*



Hemisphere.

Cut a sphere (a round apple, for instance) into a number of small pieces, passing the knife in each case through the centre of the sphere.

Each piece is a triangular solid having for its base a portion of the surface of the sphere, and for its altitude the radius of the sphere.

When the pieces are very numerous the base of each may be considered a plane and the solid a triangular pyramid. The volume of each pyramid is equal to the base $\times \frac{1}{3}$ altitude, and the total volume of all (which is the volume of the sphere) is equal to the total surface of all the bases (which is the surface of the sphere) multiplied by $\frac{1}{3}$ altitude, that is $\frac{1}{3}$ radius.

1. *Surface* = *circumference* $\times$ *diameter* $= 4\pi r^2$.

2. *Volume* $= 4\pi r^2 \times \frac{1}{3}r = \frac{4}{3}\pi r^3$.

3. Find the surface of a globe 8 inches in diameter.

4. Find the surface of the earth, its diameter being 8000 mi. ($\pi = 3{\cdot}1416$.)

5. Find the solidity of a 10-inch globe.

6. Solidity of a cannon-ball 9 inches in diameter?

7. If a heavy sphere 3 inches in diameter be immersed in a pail full of water, how much water will run over?

8. What is the weight of a cannon-ball 12 in. in diameter, iron being 7·5 times as heavy as water?

9. Find the ratio between the volume of a sphere 1 ft. in diameter, and that of a cube whose side is 1 ft.

10. Find the weight of water in a hollow sphere whose internal diameter is 3 ft. 6 in.

11. A cube and a sphere being of equal volume, find the ratio of the radius of the sphere to the side of the cube.

12. What is the solid content of a sphere when its surface is equal to that of a circle 4 feet in diameter?

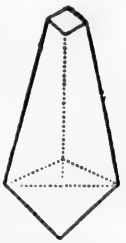
13. Find the ratio between the volume of a sphere 4 in. in diameter and that of a cylinder 4 in. high, radius of base 4 in.

14. Weight of a hollow sphere, specific gravity being 7·776, inside diameter 18 in., and thickness 2 in.?

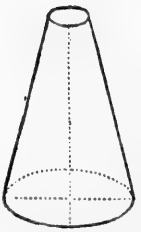
D.—The Frustum.

When the top of a cone or pyramid is cut off parallel to the base, the part remaining is called a **frustum.**

The distance between the parallel bases is called the altitude of the frustum.



Frustum of Pyramid.



Frustum of Cone.

General Rules.—(*a*) *The lateral surface of a frustum is equal to half the product of the sum of the perimeters by the slant height.*

(*b*) *The total surface of a frustum is the sum of the areas of the lateral surface and of the two bases.*

(*c*) *The volume of a frustum is equal to the sum of the areas of the two bases and the square root of their product, multiplied by one-third of the altitude.*

From which we derive the following formulæ :—

1. **Frustum of Pyramid.**—If b and b_1 stand for areas of the bases, and p and p_1 for perimeters of bases,

$$\textit{Lateral surface} = \tfrac{1}{2}(p+p_1) \times \textit{slant height}.$$

$$\textit{Total surface} = \tfrac{1}{2}(p+p_1) \times \textit{slant height} + (b+b_1).$$

$$\textit{Volume} = \tfrac{1}{3}h\,(b+b_1+\sqrt{bb_1}).$$

2. **Frustum of Cone.**—If r and r_1 stand for radii of bases.

$$\textit{Lateral surface} = \pi(r+r_1) \times \textit{slant height}.$$

$$\textit{Total surface} = \pi(r+r_1) \times \textit{slant height} + \pi(r^2+r_1^2).$$

$$\textit{Volume} = \tfrac{1}{3}\pi h\,(r^2+r_1^2,+rr_1).$$

3. The altitude of a frustum of a pyramid is 27 ft., the ends are 5 ft. and 3 ft. square; what is its solidity?

4. Find the lateral surface of the frustum of a square pyramid, one side of the upper base measuring 2 feet, of the lower base 3 feet, and having a slant height of 4 feet. Find the entire surface.

5. Draw the pattern of a small shade for a candle. Make the upper opening $1\frac{1}{2}$ inches in diameter, the lower $2\frac{1}{2}$ inches in diameter, and the slant height 2 inches. Find its convex surface.

6. How many square inches of tin will be required to make a circular pan (open at the top), its diameter at top being 9 inches, at bottom 6 inches, and slant height 4 inches. Draw the pattern.

7. Find the cost of a piece of marble in the form of the frustum of a cone, the diameters of the two ends being 11 ft. 8 in. and 8 ft. 2 in., and the slant height 18 ft. 5 in., and the value of a cubic foot of marble being 12s.

8. Find the convex surface of the frustum of a cone, the circumferences of the bases being 15 inches and 20 inches, and the slant height 10 inches.

9. How many square yards are there in the entire surface of a frustum of a cone, the radius of the upper base being 3 yards, of the lower base 5 yards, and the slant height 6 yards?

10. Find the volume of the frustum of a cone, the circumferences of whose bases are 66 and 56 feet, and whose height is 4 feet.

11. Find the weight of water in a tank in the form of a frustum of a cone, the radii of the ends being 10 feet and 8 feet, and the distance between being 5 feet.

12. How many sq. feet of tin will be required for a funnel if the diameters at top and bottom are to be 28 in. and 14 in., and the slant height 24 in.?

13. How often can a conical wine-glass, $2\frac{1}{2}$ in. in diameter and 2 in. deep, be filled from a cylindrical tumbler, 4 in. in diameter and $3\frac{1}{2}$ in. deep?

FORMULÆ IN MENSURATION.

I.—PLANE FIGURES.

If b stands for base, h for altitude, r for radius, d for diagonal, formulæ will be as follows :—

Parallelogram.

area $= bh.$

Triangle.

area $= \frac{1}{2}bh.$

Trapezoid.

area $= \frac{1}{2}(b+b_1)\times h.$

Trapezium.

area $= \frac{1}{2}(h+h_1)\times d.$

Circle.

circumference $= 2\pi r$

area $= \pi r^2$

$\sqrt{2} = 1{\cdot}414.$ $\qquad$ $\sqrt[3]{\pi} = 1{\cdot}465.$

$\sqrt{3} = 1{\cdot}732.$ $\qquad$ $\frac{1}{\pi} = {\cdot}3183.$

$\sqrt{\pi} = 1{\cdot}772.$ $\qquad$ $\frac{1}{\sqrt{\pi}} = {\cdot}5642.$

II.—SOLIDS.

If b stands for area of base, p for perimeter of base, r for radius of base, h for altitude, and l for slant height, formulæ will be as follows :

Prism.

lateral surface $= ph.$

total surface $= ph+2b.$

volume $= bh.$

Cylinder.

lateral surface $= 2\pi rh.$

total surface $= 2\pi rh+2\pi r^2.$

volume $= \pi r^2 l.$

Pyramid.

lateral surface $= \frac{1}{2}pl.$

total surface $= \frac{1}{2}pl+b.$

volume $= \frac{1}{3}bh.$

Cone.

lateral surface $= \pi rl.$

total surface $= \pi rl+\pi r^2.$

volume $= \frac{1}{3}\pi r^2 h.$

Sphere.

surface $= 4\pi r^2.$

volume $= \frac{4}{3}\pi r^3.$

Frustum of Pyramid.

lateral surface $= \frac{1}{2}(p+p_1)\times l.$

total surface $= \frac{1}{2}(p+p_1)\,l+(b+b_1).$

volume $= \frac{1}{3}h\,(b+b_1+\sqrt{bb_1}).$

Frustum of Cone.

lateral surface $= \pi l\,(r+r_1).$

total surface $= \pi l\,(r+r_1)+\pi(r^2+r_1^2).$

volume $= \frac{1}{3}\pi h\,(r^2+r_1^2+rr_1).$

XXXII.

TEST EXERCISES IN MENTAL ARITHMETIC.

A

1. 729 articles at 15 cents each.
2. 846 articles at $33\frac{1}{3}$ cents each.
3. 397 articles at $12\frac{1}{2}$ cents each.
4. 867 articles at $66\frac{2}{3}$ cents each.
5. 648 articles at $37\frac{1}{2}$ cents each.
6. 296 articles at $62\frac{1}{2}$ cents each.
7. 588 articles at $87\frac{1}{2}$ cents each.
8. 588 articles at $8\frac{1}{3}$ cents each.
9. 588 articles at $16\frac{2}{3}$ cents each.
10. 588 articles at $1.25 each.

B

1. Multiply 68 by 75.
2. Cost of $68\frac{3}{4}$ articles at 8 cents each.
3. 9 acres 90 sq. rods at $10 a sq. rod.
4. Express $\cdot 594$ as a vulgar fraction.
5. Difference between $\cdot 16$ and $\cdot \dot{1}\dot{6}$ in common fractions.
6. Interest on $\$333.33\frac{1}{3}$ for 5 mos. at 6 per cent.
7. L.C.M. of 1, 2, 3, 4, 5, 6.
8. $6\frac{1}{3}+7\frac{1}{2}+8\frac{3}{4}-21$.
9. $\{(13+7)\div 8\}\times 16=$
10. $\dfrac{(6\frac{1}{5}-3\frac{1}{6})}{6\frac{1}{3}+3\frac{1}{6}}\times 6$.

C

1. $365\times 4\frac{1}{2}=$
2. School of 85 pupils; 45 boys. Percentage of girls?
3. $\$13\frac{1}{6}\times 12\div(\frac{2}{17}\div\frac{1}{34})$.
4. $\dfrac{13\times 2\times 8\times 7\times\frac{6}{7}}{26\times 48}$.

5. Reduce 3 qts. 1 pt. to fraction of a bushel.
6. Reduce 7s. $8\frac{1}{2}$d. to fraction of £1.
7. One cent and a half a day; how much per year?
8. Amount of \$100 for 2 yrs. at $4\frac{1}{2}\%$.
9. \$9 a year; how much a day?
10. $1\frac{1}{2} : 7\frac{1}{2} :: .5 : x$.

D

1. Value of 4·15 of $\$0.33\frac{1}{3}$?
2. $\frac{3}{7} + \frac{11}{14} - 1\frac{3}{4\,9} =$
3. Rent of 5 acres at $12\frac{1}{2}$ cents a sq. rod?
4. Cost price, $\$1\frac{2}{3}$; selling price, \$2. Gain per cent.?
5. Difference between $\frac{9}{2\,5}$ and $\frac{11}{1\,2\,5}$ in decimals.
6. What decimal of $1\frac{1}{4}$ lbs. is 1 oz.?
7. Average of 13, 18, 913, $1\frac{1}{3}$, $\frac{1}{3}$, $3\frac{1}{3}$.
8. Total value of an article if $\frac{6}{11}$ is worth \$13.
9. Sum of $3\frac{1}{2}\% + 10\frac{1}{2}\%$ on \$50.
10. Number of telegraph poles 96 ft. apart in 10 miles?

E

1. Sum of all the odd numbers between 20 and 40.

2. A square contains 576 sq. inches. What is the length of a side in feet?

3. Cost of paving a yard 36 yds. long, 12 yds. wide, with asphalt 3 inches deep, at \$2 per cub. yd.?

4. A man runs 220 yards in 40 seconds. What is his rate per hour expressed in miles?

5. Number of yards of carpet, 15 inches wide, required for a room 30 feet long by $17\frac{1}{2}$ feet wide?

6. Lost 10% on \$50 and gained 12% on the remainder. Required total gain.

7. 10% lost by selling for \$65.25. Cost price = ?
8. If $\frac{1}{3} + \frac{1}{6} + \frac{1}{1\,2} =$ £2. 0s. 0d., what is the whole sum?
9. Value of 3·1 of \$15. 10. L.C.M. of 16, 24, 18.

F

1. Simple interest on $277 for 3 yrs. at 4 per cent.

2. Divide $27.30 between A and B in the ratio 9 : 4.

3. From seven thousand take fifteen hundred and 15.

4. Add five times nineteen to four times sixteen.

5. Find the fourth term of $36 : 63 :: 20 : x$.

6. Express 3s. 6d. as the decimal of £1.

7. A man rows 4 miles down the stream in 20 min., and up the stream in 40 min. What rate in smooth water?

8. Required the time for two trains to pass each other in opposite directions, each train being 220 yards long and going at the rate of 40 miles an hour.

9. A man gave $30 for 44 cwt. and sold at $\$\dot{7}\dot{2}$ per cwt. What is his gain?

10. Amount of annual income secured by investing $504 in 4 per cents. at 96?

G

1. $90 \times \frac{\cdot 06\dot{8}}{1\cdot 24}$

2. A starts at 6 mi. an hour, and in half-an-hour B follows at 9 mi. an hour. When will B overtake A?

3. The surface of a cube is 150 sq. inches; what is the length of its edge?

4. What sum at 7% will gain $84 in a year?

5. A mother is half as old again as her daughter, and their united ages = 95. Find their ages.

6. Cost of $59\frac{1}{2}$ lbs. of cheese at $9\frac{1}{2}$ cents a lb.

7. What are the 3 equal factors of 343?

8. 9 doz. hats at $\$1.33\frac{1}{3}$ each.

9. A can do a job in 4 days, B in 6 days. How long will they take together?

10. Present worth of $1080, due 5 years hence, at 4% per annum simple interest.

XXXIII.

TEST PROBLEMS.

A

1. Value of $\frac{3\frac{2}{5}}{12\frac{3}{4}}$ of $\frac{1\frac{4}{15} \text{ of } 3\frac{1}{4}}{2\frac{1}{3} \text{ of } 1\frac{2}{9}}$

and express the result as a decimal.

2. A man bought a piece of ground containing 0·316 ac. at 53¢ a sq. foot. What did he pay?

3. Find the compound interest of \$320 for $3\frac{1}{2}$ years at 3 per cent. half-yearly.

4. A tradesman marks his goods at 25% above cost, but allows a customer a discount of 12% for cash. What per cent. of profit will he make on a cash sale?

5. A rupee is worth 2s. $0\frac{1}{2}$d. and a dollar 4s. $4\frac{1}{2}$d. Find the least number of rupees which makes an exact number of dollars.

B

1. Simplify $\frac{\frac{2}{3}\frac{2}{1}+\frac{3}{2\frac{2}{9}}}{\frac{3}{4}\frac{4}{1}+\frac{5}{1\frac{1}{9}}} \div \frac{\frac{2}{3}\frac{1}{7}+\frac{8}{2\frac{2}{3}}}{\frac{1}{4}\frac{0}{3}+\frac{1}{3}\frac{3}{4}}$.

2. Add together 7695 sq. inches, ·601 of an acre, and $\frac{10}{11}$ of a sq. rod, giving answer in sq. yds. and decimal.

3. If 4 men mow 15 ac. in 10 dys. of 7 hrs., in how many dys. of $6\frac{1}{2}$ hrs. can 7 men mow $19\frac{1}{2}$ acres?

4. A man receives 4% on one-third of his capital, $4\frac{1}{2}$% on one-sixth, and 5% on the remainder. What percentage does he receive on the whole?

5. Find the difference between the compound interest on \$3840 for 3 years at 5 per cent. and the true discount on \$12,710.87 due 6 months hence at 10%.

C

1. Divide $\cdot\dot{7}23905$ by $2\cdot\dot{1}\dot{7}$ and express the difference between $1\cdot5384615$ and $\cdot076923$ as a common fraction.

2. If three pipes separately can fill a cistern in $17\frac{1}{7}$, $19\frac{1}{5}$ and $21\frac{9}{11}$ minutes, how long will they take when running together?

3. Divide £105 between A, B and C, so that as often as A gets 4s. 6d. B may get 7s. 6d., and as often as B gets 5s. C may get 6s.

4. Find by practice the value of 37° 16′ 30″ at 63 mi. 780 yds. for each degree.

5. If the 3 per cents. may be bought at $88\frac{1}{8}$, what should be the price of the 4 per cents.?

D

1. Simplify $\dfrac{\cdot071+\cdot0385}{\cdot71-\cdot007} \div \dfrac{3\cdot14153\div70}{\cdot041\times\cdot015}$.

2. At what time between 2 and 3 o'clock will the hands of a watch intercept an angle of 90°?

3. Find the time in the following capitals when Greenwich mean time denotes 12 noon:—St. Petersburg 30° 20′ East longitude; Berlin 13° 24′ East; Dublin 6° 16′ West; New York 73° 58′ West. (See Book III., p. 35.)

4. If the discount on a bill due five months hence at $3\frac{3}{4}$ per cent. is \$774$\frac{3}{8}$, what is the amount of the bill?

5. How many tiles 5 inches square would be required to line the bottom and sides of a swimming-bath which is 100 ft. by 30 ft. and 5 ft. deep?

E

1. Express $\sqrt{\frac{\cdot648\times\cdot901}{\cdot0234}}$ correctly to the nearest integer.

2. A and B are in partnership. A invests \$4700 for 12 months and receives \$446.50 as his share of the profits. What should B receive who invests \$2300 for 9 months?

3. Find change of income from transferring \$5400 stock from the $4\frac{2}{3}$ per cents. at 98 to the $3\frac{3}{4}$ per cents. at 75.

4. A straight plank is $3\frac{1}{2}$ inches thick and $6\frac{1}{4}$ inches broad; what length should be cut off so as to contain $6\frac{1}{4}$ cubic feet of timber?

5. Find the surface and diagonal of a cube of granite containing 162,144 cub. inches.

F

1. Reduce 120 grams to ounces Av.; $63\frac{3}{4}$ yds. to m.

2. A grocer buys sugar at 18¢ a kg., and sells it at one cent. per 50 grams. Find his gain per cent.

3. When gold is quoted at $\$1.12\frac{1}{2}$, what is a \$1 greenback worth?

4. What income should I derive from an investment of \$6990 in 3% stock at $87\frac{3}{8}$? How much must be invested in 4% stock at 112 to yield an equal sum?

5. The distance between two towns is 54 miles, and the distance between their places on a map is $6\frac{3}{4}$ inches. What area of country is represented by a circle on a map of one inch radius?

G

1. Find the sum of $3\frac{1}{4}$, $6\frac{5}{7}$, $7\frac{9}{11}$, $65\frac{8}{9}$; express the result as a decimal and extract the cube root.

2. How many litres in 6 gallons of water?

3. If a gallon contains $277\frac{1}{4}$ cub. inches, find approximately the number of gallons of water which would cover a square mile to the depth of an inch.

4. If 8000 metres are equivalent to 5 miles, and if a cubic fathom of water weighs 6 tons (2240 lbs.), and a cubic metre of water 1000 kg., find the number of lbs. Av. in a kilogram.

5. If a room is 40 feet long by 20 feet broad, and contains 12,800 cubic feet, what addition will be made to its cubic contents by throwing out a semicircular bow at one end? ($\pi = 3{\cdot}1416$.)

WEIGHTS AND MEASURES TABLES.

Time.

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (dy.)
7 days = 1 week (wk.)
365 days = 1 common year (yr.)
100 years = 1 century (C.)

Capacity.

2 pints (pt.) = 1 quart (qt.)
4 quarts = 1 gallon (gal.)

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

1 pint (water) weighs 1¼ lbs. Av.
1 gallon contains 277·274 cubic inches, and weighs 10 lbs. Av.
1 cubic foot (water) contains 6¼ gallons (nearly), and weighs 1000 oz. or 62½ lbs. Av.

Weight.

16 ounces (oz.) = 1 pound (lb.)
100 pounds = 1 hundredweight (cwt.)
20 cwt. or 2000 lbs. = 1 ton.
2240 lbs. = 1 long ton.

English Money.

4 farthings = 1 penny (d.)
12 pence = 1 shilling (s.)
20 shillings = 1 pound (£.)

Length.

12 inches (in.) = 1 foot (ft.)
3 feet = 1 yard (yd.)
5½ yards = 1 rod (rd.)
320 rods, 1760 yards or 5280 feet = 1 mile (mi.)

100 links = 1 chain.
80 chains = 1 mile.

Surface.

144 square inches = 1 sq. foot.
9 square feet = 1 square yard.
30¼ square yards = 1 square rod.
160 square rods or 4840 square yards = 1 acre (ac.)
640 acres = 1 square mile.

10,000 square links = 1 sq. chain.
10 square chains = 1 acre.

Volume.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.

16 cubic feet = 1 cord foot.
8 cord ft. or 128 cubic ft. = 1 cord.

Miscellaneous.

12 units = 1 dozen.
12 dozen = 1 gross.
20 units = 1 score.

24 sheets = 1 quire.
20 quires = 1 ream.

4 inches = 1 hand.
6 feet = 1 fathom.

Troy Weight.

24 grains = 1 pennyweight (dwt.)
20 dwt. = 1 ounce.
12 ounces = 1 pound.

1 lb. Troy = 5760 grains.
1 lb. Av. = 7000 grains.

Circular Measure.

60 seconds (″) = 1 minute (′)
60 minutes = 1 degree (°).
360 degrees = 1 circumference.
69⅛ miles = 1 degree.

METRIC SYSTEM.

Money.

100 centimes = 1 franc (fr.)

A franc weighs 5 grams.

Length.

10 **millimetres** (mm.) = 1 **centimetre** (cm.)
10 centimetres = 1 decimetre (dm.)
10 decimetres = 1 **metre** (m.)
10 metres = 1 decametre (Dm.)
10 decametres = 1 hectometre (Hm.)
10 hectometres = 1 **kilometre** (Km.)
10 kilometres = 1 myriametre (Mm.)

Capacity.

10 millilitres (ml.) = 1 centilitre (cl.)
10 centilitres = 1 decilitre (dl.)
10 decilitres = 1 **litre** (l.)
10 litres = 1 decalitre (Dl.)
10 decalitres = 1 **hectolitre** (Hl.)

Weight.

10 milligrams (mg.) = 1 centigram (cg.)
10 centigrams = 1 decigram (dg.)
10 decigrams = 1 **gram** (g.)
10 grams = 1 decagram (Dg.)
10 decagrams = 1 hectogram (Hg.)
10 hectograms = 1 **kilogram** (Kg.)
1000 kilograms make a metric ton.

Surface Measure.

100 square millimetres = 1 **square centimetre** (sq. cm.)
100 square centimetres = 1 square decimetre (sq. dm.)
100 square decimetres = 1 **square metre** (sq. m.)

Land Measure.

100 centiares (ca.) = 1 **are** (a.) 100 ares = 1 **hectare** (Ha.)

A centiare is the same in size as a sq. metre.

Solid Measure.

1000 cubic millimetres = 1 **cubic centimetre** (cu. cm.)
1000 cubic centimetres = 1 cubic decimetre (cu. dm.)
1000 cubic decimetres = 1 **cubic metre** (cu. m.)

In measuring wood the cubic metre is called a **stere**.

Metric Equivalents.

1 metre	= 39·37 inches.	1 litre	= 1·76 pints.
1 kilometre	= ·6214 mile.	1 hectolitre	= 22·01 gal.
1 sq. metre	= 1·196 sq. yds.	1 gram	= 15·432 grains.
1 hectare	= 2·471 acres.	1 kilo	= 2.2046 lbs. Av.
1 cu. metre	= 1·308 cu. yds.	1 metric ton	= 1·1023 tons.

Approximate Equivalents.

Metre	= 1·1 yds.	Cu. metre	= 1·3 cu. yds.
Kilometre	= $\frac{5}{8}$ mi.	Litre	= $1\frac{3}{4}$ pints.
Sq. metre	= $1\frac{1}{5}$ sq. yds.	Hectolitre	= 22 gal.
Hectare	= $1\frac{1}{2}$ acres.	Gram	= $15\frac{1}{2}$ grains.
Cu. centimetre	= $\frac{1}{16}$ cu. inch.	Kilogram	= $2\frac{1}{5}$ lbs. Av.

XXXVI.

LOGARITHMS.

The expression $10 \times 10 \times 10 \times 1 = 1000$ is usually written $10^3 = 1000$, where 3 indicates the number of factors each equal to 10 which must successively multiply unity in order to give 1000. Each number in the latter form of the expression has a name indicative of the relation it sustains to the others. Thus 1000, derived from unity by three successive multiplications by ten, is called the natural number; 10, the repeated factor, is called the base; and 3, which indicates the number of successive factors used, is called the logarithm;—fully stated, 3 is the logarithm of the natural number 1000 to the base 10. Similarly, since $2^4 = 16$, 4 is the logarithm of the natural number 16 to the base 2.

PARTIAL TABLE OF LOGARITHMS.

Natural Numbers.	Their Logarithms.	Because
1000000	6	$1 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1000000$
100000	5	$1 \times 10 \times 10 \times 10 \times 10 \times 10 = 100000$
10000	4	$1 \times 10 \times 10 \times 10 \times 10 = 10000$
1000	3	$1 \times 10 \times 10 \times 10 = 1000$
100	2	$1 \times 10 \times 10 = 100$
10	1	$1 \times 10 = 10$
1	0	1 needs no multiplier to give 1
·1	$\bar{1}$	$\frac{1}{10} = ·1$
·01	$\bar{2}$	$\frac{1}{10 \times 10} = ·01$
·001	$\bar{3}$	$\frac{1}{10 \times 10 \times 10} = ·001$
·0001	$\bar{4}$	$\frac{1}{10 \times 10 \times 10 \times 10} = ·0001$
·00001	$\bar{5}$	$\frac{1}{10 \times 10 \times 10 \times 10 \times 10} = ·00001$
·000001	$\bar{6}$	$\frac{1}{10 \times 10 \times 10 \times 10 \times 10 \times 10} = ·000001$

An unlimited number of systems of logarithms might be used; but we shall concern ourselves only with that system whose base is 10.

In the foregoing table it is easy to see the significance of the first seven logarithms and the manner in which they are found. The consistency of the rest of the table appears when we reflect on the relation of the successive natural numbers to each other and to their logarithms, and mark the meaning of the minus sign, here, as in all logarithms, printed above the number to which it belongs.

One factor of those which compose each natural number in the table, is removed from it when we divide it by 10; consequently the number of such factors indicated by its logarithm will be diminished by one. In other words,—*dividing a natural number by ten takes one from its logarithm.* For example, $1000 = 10 \times 10 \times 10 \times 1 = 10^3$, if divided by 10 will give $\frac{10^3}{10} = 10^{3-1} = 10^2 = 100$. The logarithm of 1000 is 3, of 100 is 2. Again $\frac{1000}{10 \times 10} = \frac{10^3}{10^2} = 10^{3-2} = 10^1 = 10$, a single 10, a fact indicated by saying that the logarithm of 10 is 1. Further $\frac{1000}{10 \times 10 \times 10} = \frac{10^3}{10^3} = 10^{3-3} = 10^0 = 1$ a number which has in it no ten at all as a factor, a fact indicated by saying that the logarithm of 1 is 0.*

Pushing the division still further, $\frac{1000}{10 \times 10 \times 10 \times 10} = \frac{10^3}{10^4} = 10^{3-4} = 10^{-1} = \frac{1}{10} = .1$, we reach a number which requires to be multiplied by 10 in order to be brought up to unity; it is one factor short of being unity; its logarithm is 1 less than the logarithm of 1, one less than 0; this fact is indicated by writing a minus sign over 1,

* That 1 contains no factor of any sort, whether 10 or 2 or 7 or any other is expressed by saying $10^0 = 1$, $2^0 = 1$, $7^0 = 1$, etc. In fact the logarithm of 1 is 0 whatever be the base of the system of logarithms.

thus $\bar{1}$. Similarly when we say that the logarithm of ·01 is $\bar{2}$, we are merely stating the fact that ·01 has been derived from 1 by two successive *divisions* by 10; just as, on the other hand, when we say that the logarithm of 100 is 2, we state the fact that 100 is derived from 1 by two successive *multiplications* by 10.

If any two of the first seven numbers of the foregoing table be multiplied together, it is evident that the number of tens, which successively multiplying unity give the product, is equal to the sum of the numbers indicating the tens contained in each of the two numbers multiplied together. Thus, $1000 \times 100 = 10 \times 10 \times 10 \times 1 \times 10 \times 10 \times 1 = 10 \times 10 \times 10 \times 10 \times 10 \times 1$; that is, the product contains 5 tens as factors; because one of the quantities multiplied together contained 3, and the other 2 tens as factors, and the sum of 2 and 3 is 5. This fact may be thus stated,—*the logarithm of the product of two numbers equals the sum of the logarithms of the two numbers.* To revert to the preceding illustration, the logarithm of 1000 is 3, the logarithm of 100 is 2, and the logarithm of the product of 100 and 1000, that is of 100000, is 5. If proper account be taken of the signs of the logarithms the same statement applies to all the numbers in the foregoing table. It must be borne in mind that the addition of a negative number to a positive diminishes the latter, may even reduce it to zero, or, indeed to a negative quantity. So while the sum of 2 and 3 is 5 the sum of -2 and 3 is but 1, of -2 and 2 is 0, and of -3 and 2 is -1. So the logarithm of 1000×100 is the sum of 3 and 2, 5, as before shown; the logarithm of $\cdot 01 \times 1000$ is the sum of -2 and 3, 1; of $\cdot 01 \times 100$ is the sum of -2 and 2, 0; of $\cdot 001$ and 100 is the sum of -3 and 2, -1. All this is easily verified and understood by inspection.

It is evident that the statement above made of two quantities multiplied together, may be extended to any number of quantities multiplied together, and the statement may be made and remembered in this more comprehensive form,—*the logarithm of the product of any number of factors equals the sum of the logarithms of the factors.* Thus: What is the logarithm of 1000×100? Answer; It is the sum of the logarithm of 1000, 3, and of 100, 2. That is, it is $3+2$, 5.

1. What is the logarithm of 10×100000? of 100×10000? of $1000 \times \cdot 01$? of $100000 \times \cdot 0001$? of $\cdot 01 \times \cdot 0001$? of $100 \times 10 \times \cdot 0001$? of $\cdot 01 \times \cdot 001 \times 10$? of $\cdot 01 \times 10000 \times \cdot 001$?

If the factors are alike the logarithm of their product may be as easily obtained by multiplying the logarithm of one of them by their number as by setting down the logarithm of each and adding them together. The logarithm of 100^3 is as easily got by saying 3×2 are 6 as by saying $2+2+2=6$. It is evident that *the logarithm of a power of any number is found by multiplying the logarithm of the number by the index of the power.*

2. What is the logarithm of 10^4? of $\cdot 1^3$? of 100^2? of $\cdot 01^3$? of $\cdot 001^3 \times 100^2$? of $\cdot 01^4 \times 100^3 \times \cdot 1$? of $\cdot 01^2 \times \cdot 001^3 \times 1000^4$?

Since the logarithm of a product is the sum of the logarithms of the factors, it is evident that if the logarithm of a product and that of one of the factors be known, the logarithm of the other factor will be found by subtracting the logarithm of the given factor from the logarithm of the product. Hence *the logarithm of a quotient is the logarithm of the dividend less the logarithm of the divisor.* Because a fraction is an expression indicating the division of the numerator by the denominator, the above principle

may be thus enunciated,—*the logarithm of a fraction equals the logarithm of the numerator less the logarithm of the denominator.* Thus: What is the logarithm of $1000000 \div 100$? Answer: It is the logarithm of 1000000 less the logarithm of 100; that is, it is $6-2=4$.

3. What is the logarithm of $1000 \div 10$? of $100000 \div 100$? of $100 \div 10000$? of $\cdot 01 \div \cdot 0001$? of $100 \div \cdot 001$? of $\frac{1000}{10000}$? of $\frac{10}{1000}$? of $\frac{.001}{.0001}$? of $\frac{10}{.0001}$? of $\frac{.001}{1000}$?

Since the logarithm of the square of 100, that is of 10000 is found by doubling the logarithm of 100, obviously the logarithm of the square root of 10000 is found by halving the logarithm of 10000. So the logarithm of the square root of any number is found by dividing the logarithm of the number by 2; the logarithm of the cube root of any number is found by dividing the logarithm of that number by 3; and, generally,—*the logarithm of any root of a number is found by dividing the logarithm of that number by the index of the root.* For example: What is the logarithm of the sixth root of 1000000? Answer: It is 1; because the logarithm of 1000000 is 6, and the logarithm of its 6th root is the sixth part of 6, is 1.

4. What is the logarithm of the square root of 100? of 10000? of $\cdot 01$? of $\cdot 0001$? of the cube root of 1000? of 1000000? of $\cdot 001$? of $\cdot 000001$? of the fourth root of 10000? and of the fifth root of $\cdot 00001$?

RECAPITULATORY QUESTIONS.—When the logarithms of the numbers are known, how can we find the logarithm of the product of any number of factors? of any power of a number? of a fraction? of the quotient of any dividend divided by any divisor? of any root of a number?

Note that the logarithm of the sum or of the difference of two numbers cannot be found by any simple process from the logarithms of the numbers themselves.

The table given above may be used not merely to find the logarithms of products, quotients, powers and roots, but the products, quotients, powers and roots themselves. For when the logarithms are found they will serve to point out in the table the numbers of which they are the logarithms; so indicating the answer sought. For example, if I wish to know what the quotient of 1000 times ·01 divided by the cube root of ·001 is, I may find the logarithm of the result first, which I know, from what has gone before, to be the logarithm of 1000 + the logarithm of ·01 − one third of the logarithm of ·001. This will give $3-2-(\frac{-3}{3})=3-2+1=2$. But 2 is the logarithm of 100; therefore 100 is the answer sought.

5. Solve all previous examples, substituting the word "value" for the word "logarithm" in each question. Thus the first question will read: What is the value of 1000×100? of 10×100000? etc. Many of the questions can be solved more easily without reference to logarithms, but the value of the exercises will appear later.

6. By the table of logarithms given on page 78 find the value of $1000\times\cdot1$, of $10\times100\times1000\times\cdot0001$, of $10\div\cdot001$, of $\cdot001\div\cdot00001$, of $\cdot01\times\cdot001\div100$, of $\sqrt{10000}$, of $\sqrt[3]{\cdot000001}$, of $\sqrt[3]{10}\times\sqrt[3]{100}$, of $10\sqrt{\cdot1}\div\sqrt[4]{\cdot01}$, of $\dfrac{\sqrt[3]{10}\times\sqrt{1000}}{\sqrt[6]{\cdot1}}$, of $\dfrac{\sqrt{\cdot1}\times\sqrt[8]{\cdot01}}{\sqrt[4]{\cdot001}}$, of $\sqrt[3]{10\sqrt[4]{100}}\div\dfrac{10}{\sqrt{10}}$.

Although the exercises just given, which appear difficult to one unaccustomed to the manipulation of surds, can all be readily solved by the preceding table, yet the table is much too small to be of practical value, and must be replaced by others of much greater extent.

It does not fall within the scope of this little work to show how the logarithms of such numbers as 2, 3, 5, 7, 11,

in short of all prime numbers, are calculated; but, these being calculated, it is easily seen that the logarithms of all numbers that are composed of these and of ten, in any combinations of multiplication and division, can be found. Thus, if the logarithm of 2 were known, the logarithm of 5 could be easily found by subtracting the logarithm of 2 from the logarithm of 10, which is 1; for $\frac{10}{2}=5$. The logarithms of 4, of 8, of 16, would be respectively twice, three times and four times the logarithm of 2; for $2^2=4$, $2^3=8$, and $2^4=16$. The logarithms of 20, 200, 2000, etc., would result from the logarithm of 2 by adding to it 1, 2, 3, etc.; for these are the logarithms of 10, 100 and 1000, the respective multipliers of 2 in each case. In like manner the logarithms of ·2, ·02, ·002, etc., would be found by subtracting, from the logarithm of 2, 1, 2, 3, etc.; for these are the logarithms of the divisors 10, 100, 1000, etc., by which ·2, ·02 and ·002 are derived from 2.

Because 2, 3, 7 lie between 1 and 10, and the logarithms of 1 and 10 are 0 and 1, the logarithms of 2, 3 and 7 lie somewhere between 0 and 1; that is they must be proper fractions, or non-terminating, non-recurring decimals within these limits. They are indeed non-terminating, non-recurring decimals, as is the case with all logarithms to the base 10, except 10 and its integral powers. Consequently the logarithms of all other numbers than those last mentioned are, and can be, only approximately given.

7. The logarithm of 2 is approximately ·30103. Find the logarithm of 5, of 4, of 20, of 25, of 200, of 32, of 3·2, of 8, of 250, of 50, of 512, of 51·2, of 5·12, of 125, of 1·25, of 400.

The logarithm of 2 being as given, the logarithm of ·2 which is the tenth part of 2, is equal to the logarithm of 2 less the logarithm of 10; the logarithm of ·2 then equals

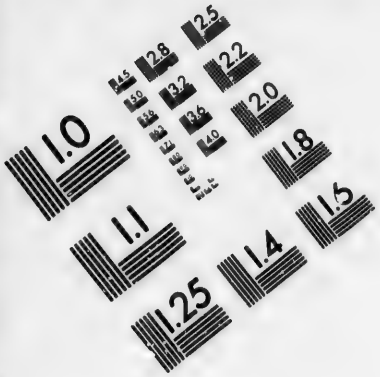
·30103 − 1. This might of course be written as −.69897; but it is found more convenient in many ways to make changes of sign only in the integral part of the logarithm, and so the logarithm of ·2 is written $\bar{1}$·30103, in which the integral part alone is negative, the decimal part remaining positive. In like manner the logarithm of ·02 which is the result of dividing 2 by 100, and of which, consequently, the logarithm is ·30103 − 2 is written as $\bar{2}$·30103. Subjoined is a table of logarithms which will prove instructive. The integral part of each logarithm is called its characteristic, and the decimal part is called its mantissa.

Natural Number.	Logarithm.
2000000	6·30103
200000	5·30103
20000	4·30103
2000	3·30103
200	2·30103
20	1·30103
2	·30103
·2	$\bar{1}$·30103
·02	$\bar{2}$·30103
·002	$\bar{3}$·30103
·0002	$\bar{4}$·30103
·00002	$\bar{5}$·30103
·000002	$\bar{6}$·30103

It will be observed :—

1st. That the numbers differ from one another only in the position of the decimal point.

2nd. That the mantissa of the logarithm remains the same throughout, notwithstanding the change of position of the decimal point in the natural numbers. This is always true of common logarithms; no change of position of the decimal point changes the mantissa.



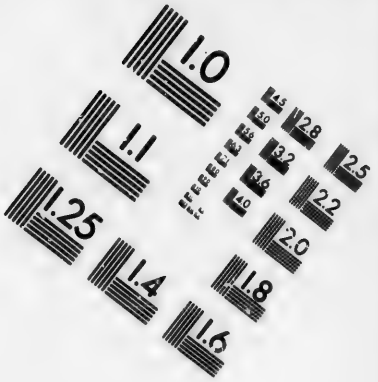
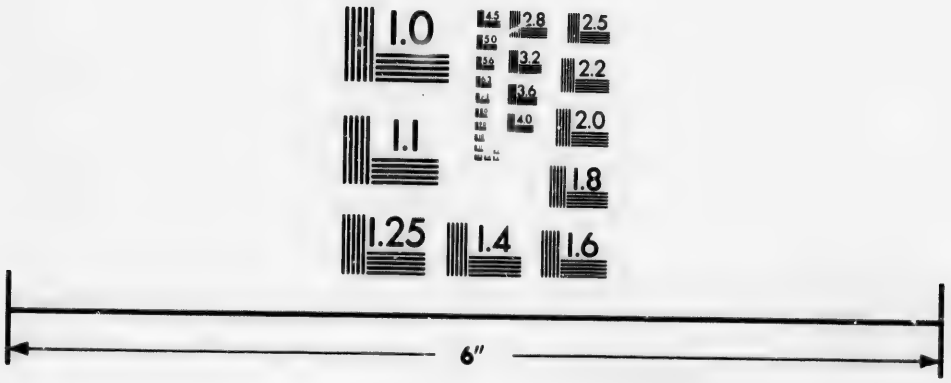
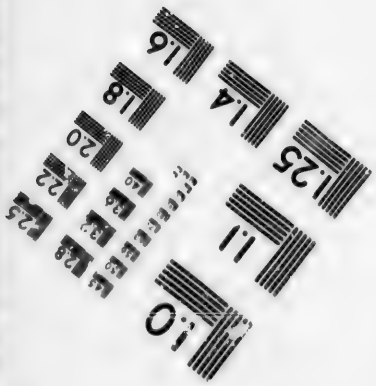
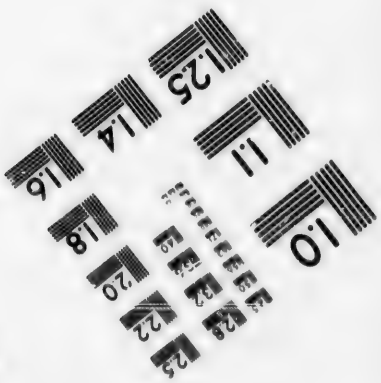


IMAGE EVALUATION TEST TARGET (MT-3)







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2.8
2.5
3.2
2.2
3.6
2.0
1.8
1.0

3rd. The characteristic changes with every change of position of the decimal point in the number; every movement of the decimal point one place further to the right diminishes the characteristic by 1. This is in accordance with what we have already learned.

4th. The characteristic indicates in each case the distance of the 2 from the units place, being positive when the 2 is to the left and negative when it is to the right of the units place.

If in any numbers the digits follow in the same order, the mantissa of the logarithms of all the numbers is the same; the logarithms will differ only in the characteristic. Thus the logarithms of 1009035, 10·09035, ·0001009035, 1009·035 are 6·00390625, 1·00390625, $\bar{4}$·00390625 and 3·00390625. The numbers differ from one another in the position of the decimal point only, and the logarithms differ in characteristics only.

The characteristic, indeed, *indicates the distance of the highest place in the number from the units place, the characteristic being positive if that highest place be to the left of the decimal point, and being negative if that highest place be to the right of the decimal point.*

Thus in the example just given 1 occupies the highest place in all the numbers. In the first number it is in the sixth place to the left of the units place, accordingly 6 is the characteristic of the logarithm; in the second instance 1 is in the first place to the left of the unit and the characteristic of its logarithm is 1; in the third instance 1 is in the fourth place to the right of the units place and the characteristic is $\bar{4}$; finally in the last instance 1 is in the third place to the left of the units place, and the characteristic of the logarithm is 3.

Hence, *to find the characteristic of the logarithm of any*

number, count from the units place to the place of the highest figure in the number, the number got by this count will be the characteristic; the characteristic will be positive if you have to count towards the left, negative if you have to count towards the right.

Examples.—Find the characteristic of the logarithm of 743·6859. Since 7 is of the highest denomination in the number, and is in the second place to the left of the units place, 2 is the characteristic. Find the characteristic of ·001392. It is $\bar{3}$, because 1, the figure of greatest value in the number, is in the third place to the right of the units place.

8. Find the characteristic of the logarithms of 49·73, 8654·227, ·3987, ·000642, ·0007, ·00892, 1·7368, 1·0009, 9·538, 9000, ·000008, ·6271.

As the characteristic of the logarithm of any number is so easily reckoned, it is not usually printed in tables of logarithms; but it is printed in the table of logarithms, page 109, which gives the logarithms of all integers from 1 to 100. Thus the logarithm of 59 is given as 1·770852, where 1 is the characteristic and ·770852 is the mantissa of the logarithm.

Examples on this First Table.

Multiplication.—Multiply 17 by 3. Process: Add the logarithms of 17 and 3 together and find of what number the result is the logarithm.

Logarithm of 17.....	1·230449
" " 3.....	·477121
The sum.....	1·707570

is the logarithm of 51, which is the product of 17 and 3.

9. By the logarithmic table, page 109, find the value of

3×7, 5×9, 7×8, $2\times3\times7$, $5\times9\times2$, $3\times3\times8$, $11\times2\times3$ and $2\times7\times7$.

DIVISION.—Divide 99 by 11. Process: Subtract the logarithm of 11 from the logarithm of 99, the remainder will be the logarithm of the quotient.

Logarithm of 99.....1·995635
" " 11.....1·041393
Difference..... ·954242

which is very nearly the logarithm of 9, the answer.

Here it will be well to note, as was before stated, that the logarithms are at best only approximately correct, and, consequently, some little doubt always hangs over the last figure of the logarithms which we find.

10. Divide by this table 21 by 7, 94 by 47, 68 by 17, 95 by 5 and 98 by 2.

INVOLUTION.—Raise 3 to the 4th power. Process: Multiply the logarithm of 3 by 4, the result will be the logarithm of the answer.

Logarithm of 3..... ·477121
4
1·908484

which is very nearly the logarithm of 81, the answer.

11. Find the square of 7, the cube of 4, the square of 6, the fifth power of 2.

EVOLUTION.—Find the sixth root of 64. Process: Divide the logarithm of 64 by 6 the quotient will be the logarithm of the answer.

Log. 64, 1·806180 divided by 6 is ·301030.

But this is the logarithm of 2, the answer.

12. Find the square root of 64, the cube root of 64 and the fifth root of 32.

Two or more of these processes may be combined in one

exercise. Thus, find the square of the cube root of the thirtieth part of 35 times 14 times 21.

Logarithm of 35.....	1·544068
14.....	1·146128
21.....	1·322219
	4·012415
30.....	1·477121
	3)2·535294
	·845098
	2
	1·690196

which is the logarithm of 49, the answer.

13. Divide 9 times 49 by 21. Multiply one seventy-fifth of 95 by 15 times 3. Multiply one third of one eleventh of 9 by one third of 22. What is the product of $1\frac{2}{3}$, $2\frac{4}{7}$, $6\frac{1}{8}$ and $1\frac{1}{11}\frac{1}{5}$? What is the cube root of the fourth power of 27? What is the cube root of the square of five sevenths of the product of $1\frac{9}{11}\frac{9}{5} \times 2\frac{4}{5} \times 2\frac{4}{11}$?

The table of logarithms of numbers from one to one hundred is too small to be of much practical value; but the table which is contained in the next 15 pages, pp. 110–124, is much more serviceable. It gives the mantissa corresponding to every possible sequence of four digits, that is, of all numbers from 1000 to 9999. The three highest digits of the number whose logarithm is required are printed in the column headed N, and the fourth digit in the same line as N at the top of the page. The corresponding mantissa, the decimal point of which is omitted in printing, is found opposite the first three digits and under the fourth digit.

Thus, if we wish to find the mantissa corresponding to 7945 we shall find on page 121, rather more than half-way

down the column N, the three first digits 794; opposite these digits, under 5, at the top of the page, we have 900094; this number, when the decimal point is supplied, ·900094, is the mantissa corresponding to the digits 7945. Again, on page 116, in column N, about one fourth of the way down, we have 473. From 473 trace the line across the page till we come under 8 at the top of the page. Here we find the number 5595; but 67, printed above, are the first figures of the mantissa, so that the mantissa corresponding to 4738 is ·675595.

Hence, to find the mantissa corresponding to any sequence of four digits, find the first three digits in one of the columns headed N, and trace a line from them across the page until under the fourth digit as printed at the top of the page; the number so found is the mantissa, which, however, must have a decimal point supplied at its left-hand side.

14. Find the mantissæ corresponding to the sequences: 3928, 4765, 9827, 5903, 4790, 6800, 7495, 5300, 7370, 1952 and 1007.

15. Bearing in mind that the mantissa is the same for the same sequence of digits, no matter where the decimal point may be, find the mantissæ for 469., 46.91, ·04691, 8800, 880, 88, 8.8, ·88, ·088, ·0088, 7000, 700, 70, 7, 5·3, ·0867, 954000, 48·76, 9·482, ·0016, 1000, 100, 10, 1, ·1, ·01, ·001, ·0001, ·5408, ·000673, ·09821.

16. By supplying the characteristic before each mantissa find the logarithms of the preceding 31 numbers.

17. Find the logarithms of 28.37, ·9546, ·48, 5·694, 7·862, 97·34, 428·7, ·006832, 59·71, ·05982, ·5907, ·00583, ·062, ·07, ·8, 95·42, 968·3, 79540, 146800, 967000.

To find the sequence of digits corresponding to a given mantissa it is, of course, necessary to reverse the process

just described. For example, to find the four digits corresponding to the mantissa ·860518, the mantissa given will be found near the middle of page 120, under 3 and opposite 725; therefore 7253 is the group of digits required.

18. What sequences correspond to the mantissæ, ·192289, ·415808, ·563718, ·700444, ·789369, ·873262, ·903090 and ·937819.

If the whole logarithm be given, characteristic as well as mantissa, the natural number corresponding to it can be found. For the mantissa will give the sequence of four digits, and the characteristic will determine the position of the decimal point among them. Thus to the logarithm 4·823018 corresponds the number 66530; for the mantissa ·823018 determines the four digits 6653, and the characteristic 4 shows that the highest figure is 4 places to the left of the units place.

19. Find the natural numbers corresponding to the logarithms 5·763428, 4·830011, 3·906443, 2·970858, 1·189209, ·330819, $\bar{1}$·440594, $\bar{2}$·527243, $\bar{3}$·560385, $\bar{4}$·597476, $\bar{5}$·635081.

In actual work two difficulties which have not yet been discussed will be encountered. First, the table gives mantissæ corresponding to four successive digits only; how shall the mantissa corresponding to five or six successive digits be found? Secondly, how is the natural number corresponding to a mantissa not in the table to be found?

Both difficulties are met by the application of the principle that when numbers, and consequently their logarithms also, are near one another, the differences between the logarithms are nearly proportionate to the differences between the numbers. This may be illustrated by refer-

ence to the table as given. Thus, 9984, 9987 and 9991 are three numbers whose successive differences, 3 and 4, are small in comparison with the numbers themselves. Now, the logarithms of these numbers are respectively 3·999305, 3·999435 and 3·999609, of which the successive differences are ·000130 and ·000174. But as 3:4::·000130: ·000173$\frac{1}{3}$, which is very nearly the same as ·000174.

The smaller relatively the differences are, the more nearly will the proportion give the correct result, and as the differences in practice are always relatively less than those in the illustration, the approximation is always nearer than in the illustration.

The proportion asserted may be applied in two ways; 1st, to find the mantissa of the logarithm of some number not found in the table; or 2nd, to find the number corresponding to a logarithm of which the mantissa is not found in the table.

Thus, 1st, if we need the logarithm of 598732, the nearest numbers to this of which we can find the logarithms from the table are 598700 and 598800. The logarithms of these are 5·777209 and 5·777282. Now, the difference between the numbers 598700 and 598800 is 100, and the difference between their logarithms is ·000073; also the difference between the numbers 598700 and 598732 is 32. Therefore by the principle stated above

100:32::·000073: the difference between the
32 logarithms of 598700
·000146 and 598732.
·00219

100)·002336(·00002336

Therefore the logarithm of 598732 equals the logarithm of 598700 increased by ·00002336, *i.e.*, 5·777209+

·00002336, *i.e.*, 5·77723226. However, as we do not know from the table the logarithm of 598700 to more than 6 decimal places, it is a mere affectation of accuracy to carry the logarithm of 598732 further, so that its logarithm right to the sixth place is 5·777232.

The labour of finding the difference between the logarithms of 598700 and 598800 is saved by the table. Under the heading D, which stands for difference, the differences between the successive mantissæ are given, So, opposite the line in which the logarithms of 598700 and 598800 are found, the difference, 73, standing for ·000073, is printed under D; and, instead of making the subtraction for ourselves, we might have copied the difference from the table.

Again, as this difference is the difference between two successive mantissæ, it is the difference in logarithms corresponding to a difference of 1 in the lowest of the 4 places for which the mantissæ are calculated in the table. Therefore one tenth of this, 7·3, would be the difference of mantissæ corresponding to a difference of one in the lowest of five successive digits, and twice, three times, four times, etc., this 7·3 would be the difference corresponding to the difference of 2, 3, 4, etc., in this fifth place of the natural number. In like manner ·73 would be the difference in mantissæ corresponding to a difference of one in the lowest of 6 digits. The difference thus corresponding to a difference of 3 in the fifth place would be $3 \times 7{\cdot}3 = 21{\cdot}9$, and of 2 in the sixth place would be $2 \times {\cdot}73 = 1{\cdot}46$; so that the difference of 32 between 598700 and 598732 would correspond to a difference between their logarithms of $21{\cdot}9 + 1{\cdot}46 = 23{\cdot}36$, it being understood that 23 is of the same denomination as the 73 from which it was derived, namely, so many millionths of unity. The logarithm of

598732, then, would be as before $5{\cdot}777209 + {\cdot}000023{\cdot}36 = 5{\cdot}777232$, omitting the last two figures as useless, because they transcend the limits of accuracy of the table as given.

A table of the amounts to be added to the mantissæ for each possible difference in the fifth place of the natural number, counting from its highest figure, when the difference between the successive mantissæ given in the table is 73, follows.

Difference in the fifth place of the natural numbers.	Corresponding difference of mantissæ.	Nearest number within the limit of the table as calculated.
1	7·3	7
2	14·6	15
3	21·9	22
4	29·2	29
5	36·5	37
6	43·8	44
7	51·1	51
8	58·4	58
9	65·7	66

With such a table before us, the calculation of the amount to be added to the mantissa of 5987 in order to get the mantissa of 598732 would be extremely easy. The amount corresponding to the additional 3 would be found in the table as 22; that corresponding to the 2 one place lower down would be found from the table to be the tenth part of 15, *i.e.*, would be one, and the total amount to be added for 32 would be 23 as before found.

Such a table, modified to bring it within the limits of the table as calculated, is found in the first column of the table of mantissæ. The values given above are found on page 118 under PP, which stands for proportional parts. opposite the 1, 2, 3, 4, etc., printed under 590 in the column headed 1.

Three ways of extending the table to include more than four digits in the natural number have been discussed. They all agree in this, that they furnish a method of calculating a correction which, added to the mantissa of the highest four digits of the natural number, shall give the mantissa for one or at the utmost two additional digits.

1. The first method of calculating the correction is to use directly the principle on which all three methods are founded, viz.; as the difference between two numbers near together is to the difference between one of these numbers and a third near it in value; so is the difference between the logarithms of the first two numbers to the difference between the logarithms of the second two numbers, which is the correction required.

Here remark that the first two numbers whose difference is the first term in the proportion, must be natural numbers in the table, for their logarithms must be known in order that their difference may be known, as it is required for the third term of the proportion. Remark also that they must be as near the number whose logarithm is required as possible; for the principle employed is only approximately true, and is more nearly exact the nearer the numbers dealt with are to one another. Hence it is best to take the numbers in the table that differ only in their lowest or fourth digit by unity, one being less and the other greater than the number of which the logarithm is required.

As an additional illustration let us discuss the question of what correction must be added to the mantissa for 7981 to give the mantissa for 79812345. As the mantissa remains the same, wherever the decimal point may be, we will try to find the mantissa corresponding to 7981·2345. The number whose logarithm we can find in the table

next less than 7981·2345 is 7981, of which the logarithm is 3·902057 ; and the number next greater than it is 7982, of which the logarithm is 3·902112. Therefore by our principle

7982 − 7981 : 7981·2345 − 7891 : : 3·902112 − 3·902057 : the logarithm of 7981·2345 − 3·902057 ;

that is 1 : ·2345 : : ·000055 : the correction required.

·000055
11725
11725
·0000128975 the correction required.

If the logarithms used in the calculation were exact the correction so found might be used to the last decimal; but as the logarithms given in the table are only correct to the sixth decimal place, the correction is of value only to the same extent, and is, therefore, ·000013, which, though a little too great, is nearer the calculated correction than ·000012 would be. So then the logarithm of 7981·2345, as nearly as these tables will give it, is 3·902057 + ·000013 = 3·902070.

If in the same way we found the logarithm of 7981·234 our proportion would be

1 : ·234 : : ·000055 : the correction required.

·000055
1170
1170
·000012870

which is practically the same correction as before. Again the correction needed for 7981·23 similarly found would be ·00001265, which to the sixth decimal place is the same correction as before. We may conclude that our table as given will not enable us to distinguish the logarithms of

series of digits extending to more than 6 in number. In other words, up to six places of decimals the logarithm of 57328·4659, for instance, is the same as the logarithm of 57328·4862. All digits after the 4 may be cancelled, as they will not affect the logarithms to be used, when as in the tables before us they are not calculated beyond the sixth decimal place. As, however, 57328·5 is nearer to both the numbers given than 57328·4 would be, the logarithm of 57328·5 would be found as the nearest practical representative of the logarithms of 57328·4659 or of 57328·4862. In actual work, of course, the difference between the mantissæ in the table is not written down with a long array of ciphers as in the example just worked, ·000055, but more simply 55; and the work to be done is simplified into multiplying 55 by the difference of the numbers, ·2345, so that the correction is thus found:

$$\begin{array}{r} \cdot 2345 \\ 55 \\ \hline 11725 \\ 11725 \\ \hline 12\cdot 8975 = 13 \text{ practically,} \end{array}$$

which of course is added in its proper place at the end of the mantissa of 7981.

20. Find the mantissæ corresponding to the following sequences of digits: 13796, 13847, 265778, 394876, 429948, 568872, 5688723, 5688724, 5688719, 56887198, 93462, 789546, 597989.

21. Find the logarithms of 3·6871, 45·382, ·0067935, ·0798984, 958276000, 964·7399, 11·7184, ·0005949259, 79·8463, 2758·458.

2. The second method of calculating the correction is like the first, except that instead of subtracting one man-

tissa from the next to find the difference which is the second term of the proportion, the difference is taken from the last column on the page headed D. This method is not quite so exact as the first method, because the differences in a few cases are not quite right. Thus the difference between the mantissæ corresponding to 1061 and 1062 is 410, but the difference given in the table is 408. This arises from the fact that the 408 given under D is the average difference for the line of mantissæ opposite which it is printed, and is a little too small for those at the beginning of the line and a little too great for those at its end. The error resulting from the use of the differences given in the table under D is insignificant.

It is obvious that we may find the logarithm of 736854 thus: Find the logarithm of 736800, which is 5·867350, and add to it as the correction sought the product of the tabular difference, given under D as 59, by ·54, the two digits in the fifth and sixth places. The product, $59 \times \cdot 54$, is 31·86. The nearest integer to this, 32, is to be added as the correction; it makes the logarithm of 736854 to be 5·867350+32 in the fifth and sixth places, which is 5·867382.

22. Find the mantissæ corresponding to the following sequences of digits: 357897, 682483, 795962, 7998989, 8632157, 8878981, 95342, 96878215, 9736857, 9987119.

N.B.—Since your table will not enable you to extend your sequence of digits more than to six digits, take the nearest sequence of six digits when the mantissa of a longer sequence is required. Thus, in the last two of the preceding exercises, substitute 973686 for 9736857, and 998712 for 9987119.

3. The correction for each digit in the fifth place is calculated in the column standing first on the page and

headed PP. That correction divided by 10 will be the correction for each digit in the sixth place. Accordingly the process by which the mantissa for the sequence 3791 is changed into the mantissa for 37917862, by the aid of the column of proportional parts, PP, is as follows:

Mantissa for	3791	is ·578754
Proportional part for	7	" 81
" "	8	" 93
" "	6	" 70
" "	2	" 23
Mantissa for	37917862	" ·578845023

of which the last three figures are useless, and the mantissa required to six places of decimal is ·578845.

The uselessness of going beyond the sixth place in the sequence is evident from this example also. Substitute for 37917862, 379179, which is the nearest sequence of six places to the sequence given. The mantissa for 379179 is then found as before:

Mantissa for 3791	·578754
Proportional part for 7	81
" " 9	104
Mantissa for 379179	·5788454

which, dropping the useless figure in the seventh place, is the same mantissa as for 37917862.

23. Find mantissæ for the sequences: 358462, 479193, 588677, 589734, 6181937, 62548391, 7886954, 89388172, 91384269, 9378865942.

24. Find the logarithms of 368271, 54323, 768845, 898732, ·06378145, ·0059897, ·0003682479, ·00055555, 1·003842, 1·00004.

The pupil who has mastered the preceding exercises, is able to find as correctly as the tables given will serve, the logarithm of any number, and will by similar methods be

able to use any of the more extended tables of logarithms which may be required in his work at college. It remains by an inversion of one or other of the three methods just described to find the sequence of digits corresponding to a mantissa not found in the table. So we may find the sequence of digits corresponding to the mantissa ·902070 thus: In the table find the mantissæ nearest to ·902070. These are ·902057 and ·902112, corresponding to the sequences 7981 and 7982, between which the sequence sought must lie. Then say, as the difference between the two mantissæ in the table is to the difference between the given mantissa and the one next below it in the table, so is the differences between the sequences in the table to the difference between the lower sequence taken in the table and the sequence sought. In this particular instance the proportion becomes ·902112 − ·902057 : ·902070 − ·902057 :: 1 : the correction. That is, 55 : 13 :: 1 : the correction, which is therefore $\frac{13}{55} = ·2363+$. But as these calculations cannot be relied on for more than two additional places in the sequence, and ·24 is nearer ·2363 than ·23 would be, the sequence sought may be written as 798124. By comparing this result with the inverse process on page 96 the pupil will understand more clearly how it is that the finding of sequences not in the table cannot be safely extended to more than six places in all, when a six-figure table of logarithms is used.

The method given above is an inversion of method 1 for finding the mantissa corresponding to a sequence of more than four figures. By an inversion of the second method we may find the sequence corresponding to the mantissa ·867382. The mantissa next below it in the table is ·867350, to which corresponds the sequence 7368. The difference between successive mantissæ at this part of the

table is given under D as 59. The difference between the given mantissa and the mantissa of 7368 is 32. Therefore the digits to be annexed after 7368 will be given as $\frac{3}{5}\frac{2}{9}$, that is ·542, which cannot, however, be trusted after the second figure. The corrected sequence reads 736854. Compare with the inverse work on page 98.

Finally, the development of the sequence may be effected by an inversion of the third method of correcting a mantissa, given above. Thus, to find the sequence corresponding to the the mantissa ·578845, find the nearest lower mantissa in the table, which is ·578754, corresponding to the sequence 3791. Subtract the latter from the former; the remainder is 91; the nearest proportional part below it is 81, corresponding to 7 and leaving 10; to this annex a cipher, making it 100; to this the nearest proportional part is 104, corresponding to 9. Therefore the corrected sequence is 379179.

25. In each of the three ways given above find the sequences, to six places only, corresponding to the mantissæ ·135478, ·111111, ·234567, ·345678, ·456789, ·567890, ·678901, ·789012, ·890123, ·901234. Note particularly any discrepancies of result arising from difference of method of working.

26. Of what numbers are the following the logarithms: 3·019876, 2·109876, 1·210987, 0·321098, $\bar{1}$·432109, $\bar{2}$·543210, $\bar{3}$·654321, $\bar{4}$·765432, $\bar{5}$·876543, $\bar{6}$·987654. Use the last method.

Recapitulatory Statement.—The logarithm of the product of any number of factors is the sum of the logarithms of the factors. The logarithm of a quotient is the logarithm of the dividend less the logarithm of the divisor. The logarithm of a power of a number is the logarithm of that number multiplied by the index of the power. The

logarithm of a root of a number is the logarithm of the number, divided by the index of the root.

MULTIPLICATION.

Remember that mantissæ are always positive, and that the carrying from the sum of any number of mantissæ is always positive, although some or all the characteristics may be negative.

27. What is the product of $1{\cdot}0378 \times 1{\cdot}946 \times 1{\cdot}0008 \times 3{\cdot}006 \times 5{\cdot}00082 \times 7{\cdot}48139 \times 6{\cdot}8735$; of $16{\cdot}437 \times 28{\cdot}6845 \times 357{\cdot}642 \times {\cdot}086824 \times {\cdot}00961$; of $\cdot0632 \times {\cdot}07954 \times {\cdot}068689 \times {\cdot}00074368 \times 7329{\cdot}688$; of $10084 \times 7{\cdot}3695 \times {\cdot}000086957 \times 22{\cdot}46 \times 37{\cdot}89 \times {\cdot}058276$; of $\cdot06 \times {\cdot}073 \times {\cdot}0892 \times {\cdot}6934 \times {\cdot}82659 \times {\cdot}717876$; of $179{\cdot}63 \times 482{\cdot}7 \times 795{\cdot}687 \times 555{\cdot}55 \times {\cdot}00000687324$; of $17 \times {\cdot}18 \times 19 \times {\cdot}21 \times 23 \times {\cdot}64$; of $49{\cdot}51 \times 5{\cdot}368 \times 7{\cdot}965 \times 8{\cdot}3837 \times {\cdot}0001$; of $1{\cdot}0112 \times 1{\cdot}0234 \times 1{\cdot}1378 \times 1{\cdot}6654 \times 1{\cdot}78877$; and of $637842 \times 97918 \times 654837 \times {\cdot}0079892 \times {\cdot}00004632 \times {\cdot}000005948$?

DIVISION.

When you have in the same exercise several quantities multiplied together, divided in any order by several divisors, add the logarithms of all the multipliers, and from the sum subtract the sum of the logarithms of all the divisors. In subtracting remember that your mantissæ are always positive; that taking away a negative quantity is equivalent to the addition of a positive quantity; and that taking away a negative quantity is equivalent to adding a positive quantity. Each of the points in this paragraph is illustrated by an example below.

Divide $\frac{3{\cdot}1458}{{\cdot}00673} \times \frac{7{\cdot}65}{100084}$ by $973{\cdot}84 \times 56{\cdot}673$.

Here ·00673, 100084, 973·84 and 56·673 are all divisors,

therefore add their logarithms and subtract the sum from the sum of the logarithms of 3·1458 and 7·65.

		Log. 00763.....	$\bar{3}$·882525
		" 100000.....	5·
		PP. of 80.....	331
		" 4.....	17
Log. 3·145.....	0·497621	Log. 973·8.....	2·988470
PP. 8.....	110	PP. 4.....	18
Log. 7·65	0·883661	Log. 56·67.....	1·753353
Sum.....	1·381392	PP. 3.....	23
Subtract...	7·624737	Sum.....	7·624737
	$\bar{7}$·756655	which is the logarithm of	

·000000571025, the answer.

Divide 7·6348 by ·0172.

Log. 7·634	·882752	
PP. 8.....	46	
Log. 7·6348.....	·882798	
" ·0172.....	$\bar{2}$·235528	
	2·647270	∴ log of 443·885, answer.
	2·647187	log. of 443·8
	83	
	78	PP. of 8
	50	" " 5

Divide ·0172 by 763·48.

Log. ·0172..	$\bar{2}$·235528	
" 763·48..	2·882798	
	$\bar{5}$·352730	log. of ·0000225284, answer.
	$\bar{5}$·352568	log. of ·00002252
	162	
PP. 8.....	154	
	80	
PP. 4.....	77	

Divide ·0076348 by ·0172.

Log. ·0076348.... $\bar{3}$·882798
" ·0172..... $\bar{2}$·235528
$\bar{1}$·647270 = log. ·443885, answer.

Divide ·0172 by ·0076348.

Log. ·0172..... $\bar{2}$·235528
" ·0076348.... $\bar{3}$·882798
0·352730 = log. 2·25284, answer.

Divide ·172 by ·00076348.

Log. ·172..... $\bar{1}$·235528
" ·00076348... $\bar{4}$·882798
2·352730 = log. 225·284, answer.

28. Divide 59·6834 by 47·9218, 736·68 by 795·887, 768000 by 754, 327·88 by 6742·11, ·76957 by ·38621, ·5948 by ·038797, ·008255 by ·018173, ·0006866 by 495·371, ·0068524 by ·0000798, and ·0000798 by ·0068524.

29. Find the reciprocal of 11, of 14·682, of 97·381, of 3754·68, of ·01, of ·0001, of ·0036827, of ·0088286, of 556789, of ·556789.

30. Give the value in decimals of the following fractions: $\frac{13}{10}$, $\frac{285}{367}$, $\frac{1928}{7564}$, $\frac{76285}{94829}$, $\frac{1}{3725}$, $\frac{.687}{.942}$, $\frac{.868}{.0012}$, $\frac{14.965}{17.897}$, $\frac{28.683}{359.991}$, $\frac{.0682}{.0079148}$.

31. Find the values of the reciprocals of the fractions in the preceding exercise.

32. Divide the product of $67 \times 39{\cdot}58 \times {\cdot}067879$ by the product of $112 \times 7{\cdot}8868 \times 59{\cdot}473$. Find the value in decimals of $\frac{786}{395} \times \frac{947}{1018} \times \frac{6734}{9482} \times \frac{37}{989}$. Find the value of $\frac{.3781}{1.0972} \times \frac{6.737}{14.982} \times \frac{68.88}{99.74} \div \frac{3428}{6957}$. Find the value of $\frac{5.683}{17.991} \times \frac{14}{15.687} \div \left(\frac{5.1147}{14.1183} \times \frac{154}{197.901}\right)$.

Involution.

Here observe carefully that the carrying from any mul-

tiplication of mantissæ is always positive, even although the characteristic and its product are negative. Thus, we wish to get the cube of ·9874, its logarithm is $\bar{1}$·994493 which when multiplied by 3 gives $\bar{1}$·983479, for the carrying from the multiplication of ·9 by 3 is +2 which added to the three times $\bar{1}$ gives $\bar{1}$. As $\bar{1}$·983479 is the logarithm of ·962672, this last is of course the cube of ·9874, right to the fifth decimal place.

33. Find the square of 19·682, and of ·07975, the cube of 7·6864 and of ·0777777, the fourth power of 1·897, the 5th power of 1·4158, the 13th power of ·77816, the 19th power of 1·12345, the 11th power of the square of ·02468 and the 9th power of the cube of 1·0124.

Evolution.

It will be remembered that to find the logarithm of the root of a number the logarithm of that number is to be divided by the index of the root. This is easily done when the characteristic and the mantissa of that logarithm are positive; also when the characteristic although negative is exactly divisible by the index of the root. So there will be no difficulty in finding the logarithm of the sixth root of that number whose logarithm is 5·739874, the quotient, which is the logarithm of the root, being ·956646.* Nor is there any difficulty in finding the logarithm of the sixth root of that number whose logarithm is $\bar{12}$·358726, the nearest quotient being $\bar{2}$·059788, and being the logarithm of the root sought, But if we are finding the sixth root of that number whose logarithm is $\bar{10}$·476793 we meet the difficulty that 6 is contained in $\bar{10}$, $\bar{1}$ with a remainder $\bar{4}$. How can this negative 4 be so blended with

* The pupil will notice that the last figure is a little too large, but it is nearer the truth than 5 in that place would be.

the positive mantissa that follows it, as to give a positive quotient, and this we need because every mantissa is to be positive. Perhaps at first sight the difficulty seems insuperable; but it may be readily overcome by saying that 6 goes into $\bar{1}\bar{0}$, $\bar{2}$ times with a remainder of $+2$; for, while it is quite true that if 6 be multiplied by $\bar{1}$ and the result be subtracted from $\bar{1}\bar{0}$, $\bar{4}$ will remain, it is no less true that if 6 be multiplied by $\bar{2}$, and the result be subtracted from $\bar{1}\bar{0}$, $+2$ will remain. The necessary division is thus accomplished:

$$6)\overline{10}\cdot476793$$
$$\overline{2}\cdot412799$$

the quotient being the logarithm of the answer.

One example more will suffice. What is the cube root of ·0007585? The logarithm of ·0007585 is $\bar{4}$·879956 of which the third part is found as above to be $\bar{2}$·959985. This last is the logarithm of ·091198; and this again is the cube root of ·0007585, right to the last digit.

34. Find the square root of 933156, of 8335·69, of 28·4089, of ·383161 and of ·00168921. Extract the cube root of 804357, of 50·653, of 166·375, of 9, of 574. What is the fourth root of ·938, the fifth root of ·0007, the sixth root of 4·3928, the seventh root of ·018195, the eighth root of 1·0789, the ninth root of 548·733, the tenth root of ·0078145, the one hundred and nineteenth root of 67·7349, the two hundred and fifteenth root of ·000068782, and the three hundred and seventeenth root of 115·317?

The preceding examples demonstrate the great assistance that logarithms may be in effecting intricate calculations that involve multiplications, divisions, involutions and evolutions only. It must never be forgotten that additions and subtractions of natural numbers cannot be effected logarithmically; hence additions and subtractions

must be done either before logarithms are introduced in the example, or after return to natural numbers has been made. Thus, to solve the following problem, which cannot be easily done without the aid of logarithms, the following order of operations must be followed with but slight variation :

$$\sqrt[5]{\sqrt[7]{9}-\sqrt[8]{11}}+\sqrt{\sqrt[5]{14}-\sqrt[14]{5}}.$$

$\frac{\text{Log. } 9}{7} = \frac{\cdot 954243}{7} = \cdot 136320$ which is log. of 1·36873

$\frac{\text{Log. } 11}{8} = \frac{1\cdot 041393}{11} = \cdot 094672$ " " " " 1·24358

$\sqrt[5]{1\cdot 36873 - 1\cdot 24358} = \sqrt[5]{\cdot 12515}$

$\frac{\text{Log. } \cdot 12515}{5} = \frac{\bar{1}\cdot 097431}{5} = \bar{1}\cdot 819486 =$ log. of ·659911

$\frac{\text{Log. } 14}{5} = \frac{1\cdot 146128}{5} = \cdot 229226 =$ log. of 1·69521

$\frac{\text{Log. } 5}{14} = \frac{\cdot 698970}{14} = \cdot 049926 =$ log. of 1·12185

$\sqrt[5]{14} - \sqrt[14]{5} = 1\cdot 69521 - 1\cdot 12185 = \cdot 57336$

$\frac{\text{Log. } \cdot 57336}{2} = \frac{\bar{1}\cdot 758428}{2} = \bar{1}\cdot 879214 =$ log. of ·757205

·659911 + ·757205 = 1·417116, answer.

35. Find the value of $(\sqrt[3]{987\cdot 6} - \sqrt{49\cdot 59}) \times (\sqrt[4]{1189} - \sqrt[5]{1200\cdot 68})$; of $(\sqrt{48\cdot 1} + \sqrt{37\cdot 695})(\sqrt{48\cdot 1} - \sqrt{37\cdot 695})$; of $\sqrt[4]{798\cdot 1 \times \cdot 0069 - 46\cdot 384 \times \cdot 018972}$; of $\sqrt[5]{\sqrt{95\cdot 87} \times \sqrt[3]{487\cdot 7}} - \sqrt[5]{\sqrt[4]{69\cdot 793}} - \sqrt[5]{892\cdot 45}$; and of $\left(\frac{\sqrt[3]{98732\cdot 7} - \sqrt{683\cdot 4}}{\sqrt[3]{98732\cdot 7} + \sqrt{683\cdot 4}}\right)^{15}$.

36. By logarithms solve the following examples from earlier pages: P. 5,—42, 43, 64, 67, 68, 81, 83, 86, 91, 93; p. 6,—4, 5, 7, 10, 11, 14, 18, 20, 21, 23; p. 16,—42, 43, 44, 46, 47, 48, 50, 51, 54, 61; p. 48,—25, 26, 29, 30, 31, 33, 34, 36; pp. 50–52,—13, 14, 17, 31, 33, 36, 40, 41; pp. 53, 54,—10, 11, 16, 19, 20, 28, 32, 39, 41, 43, 44; p. 57,—2, 3,

6, 7 ; p. 59,—6, 8, 9, 10 ; p. 60,—7, 8, 9, 11, 14, 16 ; pp. 64, 65,—3, 9, 10, 11, 14, 16, 18 ; p. 66,—8, 10, 11, 14.

37. What is the amount at simple interest of \$494·36 for 11 years at $4\frac{1}{7}\%$ per annum ? What would it be at compound interest ? What at compound interest is the amount of 1 mill for 500 years, at 3% per annum ? What is the compound interest of \$728·36 at $4\frac{1}{8}\%$ for 23 years ?

38. What is the acreage of a triangle whose base is 74·35 chains and altitude 49·77 chains ? of a triangle whose three sides are 56·38 chains, 49·71 chains and 52·31 chains ? of a parallelogram whose base is 38·94 chains and perpendicular height 42·17 chains ? of a rectangle whose con-terminous sides are 15·69 chains and 14·48 chains ? of a rectangle of which one side is 13·87 chains, and the diagonal 15·93 chains ? of a parallelogram of which two adjacent sides and one diagonal are respectively 13·47 chains, 19·63 chains and 16·18 chains ? of a circular enclosure of which the diameter is 9·36 chains ? of a walk one rod wide around the outside of this enclosure ?

39. If 45·37 were accurately raised to the one millionth power, how many volumes of 320 pages each with 60 lines on a page and 60 digits in a line would be required to hold the answer ?

MATHEMATICAL TABLES.

LOGARITHMS OF NUMBERS FROM 1 TO 10,000, WITH DIFFERENCES AND PROPORTIONAL PARTS.

Numbers from 1 to 100.

No.	Log.	No.	Log.	No.	Log.	No.	Log.	No.	Log.
1	0·000000	21	1·322219	41	1·612784	61	1·785330	81	1·908485
2	0·301030	22	1·342423	42	1·623249	62	1·792392	82	1·913814
3	0·477121	23	1·361728	43	1·633468	63	1·799341	83	1·919078
4	0·602060	24	1·380211	44	1·643453	64	1·806180	84	1·924279
5	0·698970	25	1·397940	45	1·653213	65	1·812913	85	1·929419
6	0·778151	26	1·414973	46	1·662758	66	1·819544	86	1·934498
7	0·845098	27	1·431364	47	1·672098	67	1·826075	87	1·939519
8	0·903090	28	1·447158	48	1·681241	68	1·832509	88	1·944483
9	0·954243	29	1·462398	49	1·690196	69	1·838849	89	1·949390
10	1·000000	30	1·477121	50	1·698970	70	1·845098	90	1·954243
11	1·041393	31	1·491362	51	1·707570	71	1·851258	91	1·959041
12	1·079181	32	1·505150	52	1·716003	72	1·857332	92	1·963788
13	1·113943	33	1·518514	53	1·724276	73	1·863323	93	1·968483
14	1·146128	34	1·531479	54	1·732394	74	1·869232	94	1·973128
15	1·176091	35	1·544068	55	1·740363	75	1·875061	95	1·977724
16	1·204120	36	1·556303	56	1·748188	76	1·880814	96	1·982271
17	1·230449	37	1·568202	57	1·755875	77	1·886491	97	1·986772
18	1·255273	38	1·579784	58	1·763428	78	1·892095	98	1·991226
19	1·278754	39	1·591065	59	1·770852	79	1·897627	99	1·995635
20	1·301030	40	1·602060	60	1·778151	80	1·903090	100	2·000000

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	100	000000	000434	000868	001301	001734	002166	002598	003029	003461	003891	432
41	1	4321	4751	5181	5609	6038	6466	6894	7321	7748	8174	428
83	2	8600	9026	9451	9876	010300	010724	011147	011570	011993	012415	424
124	3	012837	013259	013680	014100	4521	4940	5360	5779	6197	6616	420
166	4	7033	7451	7868	8284	8700	9116	9532	9947	020361	020775	416
207	5	021189	021603	022016	022428	022841	023252	023664	024075	4486	4896	412
248	6	5306	5715	6125	6533	6942	7350	7757	8164	8571	8978	408
290	7	9384	9789	030195	030600	031004	031408	031812	032216	032619	033021	404
331	8	033424	033826	4227	4628	5029	5430	5830	6230	6629	7028	400
373	9	7426	7825	8223	8620	9017	9414	9811	040207	040602	040998	397
	110	041393	041787	042182	042576	042969	043362	043755	044148	044540	044932	393
38	1	5323	5714	6105	6495	6885	7275	7664	8053	8442	8830	390
76	2	9218	9606	9993	050380	050766	051153	051538	051924	052309	052694	386
113	3	053078	053463	053846	4230	4613	4996	5378	5760	6142	6524	383
151	4	6905	7286	7666	8046	8426	8805	9185	9563	9942	060320	379
189	5	060698	061075	061452	061829	062206	062582	062958	063333	063709	4083	376
227	6	4458	4832	5206	5580	5953	6326	6699	7071	7443	7815	373
265	7	8186	8557	8928	9298	9668	070038	070407	070776	071145	071514	370
302	8	071882	072250	072617	072985	073352	3718	4085	4451	4816	5182	366
340	9	5547	5912	6276	6640	7004	7368	7731	8094	8457	8819	363
	120	079181	079543	079904	080266	080626	080987	081347	081707	082067	082426	360
35	1	082785	083144	083503	3861	4219	4576	4934	5291	5647	6004	357
70	2	6360	6716	7071	7426	7781	8136	8490	8845	9198	9552	355
104	3	9905	090258	090611	090963	091315	091667	092018	092370	092721	093071	352
139	4	093422	3772	4122	4471	4820	5169	5518	5866	6215	6562	349
174	5	6910	7257	7604	7951	8298	8644	8990	9335	9681	100026	346
209	6	100371	100715	101059	101403	101747	102091	102434	102777	103119	3462	343
244	7	3804	4146	4487	4828	5169	5510	5851	6191	6531	6871	341
278	8	7210	7549	7888	8227	8565	8903	9241	9579	9916	110253	338
313	9	110590	110926	111263	111599	111934	112270	112605	112940	113275	3609	335
	130	113943	114277	114611	114944	115278	115611	115943	116276	116608	116940	333
32	1	7271	7603	7934	8265	8595	8926	9256	9586	9915	120245	330
64	2	120574	120903	121231	121560	121888	122216	122544	122871	123198	3525	328
97	3	3852	4178	4504	4830	5156	5481	5806	6131	6456	6781	325
129	4	7105	7429	7753	8076	8399	8722	9045	9368	9690	130012	323
161	5	130334	130655	130977	131298	131619	131939	132260	132580	132900	3219	321
193	6	3539	3858	4177	4496	4814	5133	5451	5769	6086	6403	318
225	7	6721	7037	7354	7671	7987	8303	8618	8934	9249	9564	316
258	8	9879	140194	140508	140822	141136	141450	141763	142076	142389	142702	314
290	9	143015	3327	3639	3951	4263	4574	4885	5196	5507	5818	311
	140	146128	146438	146748	147058	147367	147676	147985	148294	148603	148911	309
30	1	9219	9527	9835	150142	150449	150756	151063	151370	151676	151982	307
60	2	152288	152594	152900	3205	3510	3815	4120	4424	4728	5032	305
90	3	5336	5640	5943	6246	6549	6852	7154	7457	7759	8061	303
120	4	8362	8664	8965	9266	9567	9868	160168	160469	160769	161068	301
150	5	161368	161667	161967	162266	162564	162863	3161	3460	3758	4055	299
180	6	4353	4650	4947	5244	5541	5838	6134	6430	6726	7022	297
210	7	7317	7613	7908	8203	8497	8792	9086	9380	9674	9968	295
240	8	170262	170555	170848	171141	171434	171726	172019	172311	172603	172895	293
270	9	3186	3478	3769	4060	4351	4641	4932	5222	5512	5802	291
	150	176091	176381	176670	176959	177248	177536	177825	178113	178401	178689	289
28	1	8977	9264	9552	9839	180126	180413	180699	180986	181272	181558	287
56	2	181844	182129	182415	182700	2985	3270	3555	3839	4123	4407	285
84	3	4691	4975	5259	5542	5825	6108	6391	6674	6956	7239	283
112	4	7521	7803	8084	8366	8647	8928	9209	9490	9771	190051	281
140	5	190332	190612	190892	191171	191451	191730	192010	192289	192567	2846	279
168	6	3125	3403	3681	3959	4237	4514	4792	5069	5346	5623	278
196	7	5900	6176	6453	6729	7005	7281	7556	7832	8107	8382	276
224	8	8658	8932	9206	9481	9755	200029	200303	200577	200850	201124	274
252	9	201397	201670	201943	202216	202488	2761	3033	3305	3577	3848	272

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	160	204120	204391	204663	204934	205204	205475	205746	206016	206286	206556	271
26	1	6826	7096	7365	7634	7904	8173	8441	8710	8979	9247	269
53	2	9515	9783	210051	210319	210586	210853	211121	211388	211654	211921	267
79	3	212188	212454	2720	2986	3252	3518	3783	4049	4314	4579	266
105	4	4844	5109	5373	5638	5902	6166	6430	6694	6957	7221	264
132	5	7484	7747	8010	8273	8536	8798	9060	9323	9585	9846	262
158	6	220108	220370	220631	220892	221153	221414	221675	221936	222196	222456	261
184	7	2716	2976	3236	3496	3755	4015	4274	4533	4792	5051	259
210	8	5309	5568	5826	6084	6342	6600	6858	7115	7372	7630	258
237	9	7887	8144	8400	8657	8913	9170	9426	9682	9938	230193	256
	170	230449	230704	230960	231215	231470	231724	231979	232234	232488	232742	254
25	1	2996	3250	3504	3757	4011	4264	4517	4770	5023	5276	253
50	2	5528	5781	6033	6285	6537	6789	7041	7292	7544	7795	252
74	3	8046	8297	8548	8799	9049	9299	9550	9800	240050	240300	250
99	4	240549	240799	241048	241297	241546	241795	242044	242293	2541	2790	249
124	5	3038	3286	3534	3782	4030	4277	4525	4772	5019	5266	248
149	6	5513	5759	6006	6252	6499	6745	6991	7237	7482	7728	246
174	7	7973	8219	8464	8709	8954	9198	9443	9687	9932	250176	245
198	8	250420	250664	250908	251151	251395	251638	251881	252125	252368	2610	243
223	9	2853	3096	3338	3580	3822	4064	4306	4548	4790	5031	242
	180	255273	255514	255755	255996	256237	256477	256718	256958	257198	257439	241
24	1	7679	7918	8158	8398	8637	8877	9116	9355	9594	9833	239
47	2	260071	260310	260548	260787	261025	261263	261501	261739	261976	262214	238
71	3	2451	2688	2925	3162	3399	3636	3873	4109	4346	4582	237
94	4	4818	5054	5290	5525	5761	5996	6232	6467	6702	6937	235
118	5	7172	7406	7641	7875	8110	8344	8578	8812	9046	9279	234
141	6	9513	9746	9980	270213	270446	270679	270912	271144	271377	271609	233
165	7	271842	272074	272306	2538	2770	3001	3233	3464	3696	3927	232
188	8	4158	4389	4620	4850	5081	5311	5542	5772	6002	6232	230
212	9	6462	6692	6921	7151	7380	7609	7838	8067	8296	8525	229
	190	278754	278982	279211	279439	279667	279895	280123	280351	280578	280806	228
22	1	281033	281261	281488	281715	281942	282169	2396	2622	2849	3075	227
45	2	3301	3527	3753	3979	4205	4431	4656	4882	5107	5332	226
67	3	5557	5782	6007	6232	6456	6681	6905	7130	7354	7578	225
89	4	7802	8026	8249	8473	8696	8920	9143	9366	9589	9812	223
112	5	290035	290257	290480	290702	290925	291147	291369	291591	291813	292034	222
134	6	2256	2478	2699	2920	3141	3363	3584	3804	4025	4246	221
156	7	4466	4687	4907	5127	5347	5567	5787	6007	6226	6446	220
178	8	6665	6884	7104	7323	7542	7761	7979	8198	8416	8635	219
201	9	8853	9071	9289	9507	9725	9943	300161	300378	300595	300813	218
	200	301030	301247	301464	301681	301898	302114	302331	302547	302764	302980	217
21	1	3196	3412	3628	3844	4059	4275	4491	4706	4921	5136	216
42	2	5351	5566	5781	5996	6211	6425	6639	6854	7068	7282	215
64	3	7496	7710	7924	8137	8351	8564	8778	8991	9204	9417	213
85	4	9630	9843	310056	310268	310481	310693	310906	311118	311330	311542	212
106	5	311754	311966	2177	2389	2600	2812	3023	3234	3445	3656	211
127	6	3867	4078	4289	4499	4710	4920	5130	5340	5551	5760	210
148	7	5970	6180	6390	6599	6809	7018	7227	7436	7646	7854	209
170	8	8063	8272	8481	8689	8898	9106	9314	9522	9730	9938	208
191	9	320146	320354	320562	320769	320977	321184	321391	321598	321805	322012	207
	210	322219	322426	322633	322839	323046	323252	323458	323665	323871	324077	206
20	1	4282	4488	4694	4899	5105	5310	5516	5721	5926	6131	205
40	2	6336	6541	6745	6950	7155	7359	7563	7767	7972	8176	204
61	3	8380	8583	8787	8991	9194	9398	9601	9805	330008	330211	203
81	4	330414	330617	330819	331022	331225	331427	331630	331832	2034	2236	202
101	5	2438	2640	2842	3044	3246	3447	3649	3850	4051	4253	202
121	6	4454	4655	4856	5057	5257	5458	5658	5859	6059	6260	201
141	7	6460	6660	6860	7060	7260	7459	7659	7858	8058	8257	200
162	8	8456	8656	8855	9054	9253	9451	9650	9849	340047	340246	199
182	9	340444	340642	340841	341039	341237	341435	341632	341830	2028	2225	198

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	220	342423	342620	342817	343014	343212	343409	343606	343802	343999	344196	197
19	1	4392	4589	4785	4981	5178	5374	5570	5766	5962	6157	196
39	2	6353	6549	6744	6939	7135	7330	7525	7720	7915	8110	195
58	3	8305	8500	8694	8889	9083	9278	9472	9666	9860	350054	194
77	4	350248	350442	350636	350829	351023	351216	351410	351603	351796	1989	193
97	5	2183	2375	2568	2761	2954	3147	3339	3532	3724	3916	193
116	6	4108	4301	4493	4685	4876	5068	5260	5452	5643	5834	192
135	7	6026	6217	6408	6599	6790	6981	7172	7363	7554	7744	191
154	8	7935	8125	8316	8506	8696	8886	9076	9266	9456	9646	190
174	9	9835	360025	360215	360404	360593	360783	360972	361161	361350	361539	189
	230	361728	361917	362105	362294	362482	362671	362859	363048	363236	363424	189
19	1	3612	3800	3988	4176	4363	4551	4739	4926	5113	5301	188
37	2	5488	5675	5862	6049	6236	6423	6610	6796	6983	7169	187
56	3	7356	7542	7729	7915	8101	8287	8473	8659	8845	9030	186
74	4	9216	9401	9587	9772	9958	370142	370328	370513	370698	370883	185
93	5	371068	371253	371437	371622	371806	1991	2175	2360	2544	2728	184
111	6	2912	3096	3280	3464	3647	3831	4015	4198	4382	4565	184
130	7	4748	4932	5115	5298	5481	5664	5846	6029	6212	6394	183
148	8	6577	6759	6942	7124	7306	7488	7670	7852	8034	8216	182
167	9	8398	8580	8761	8943	9124	9306	9487	9668	9849	380030	181
	240	380211	380392	380573	380754	380934	381115	381296	381476	381656	381837	181
18	1	2017	2197	2377	2557	2737	2917	3097	3277	3456	3636	180
35	2	3815	3995	4174	4353	4533	4712	4891	5070	5249	5428	179
53	3	5606	5785	5964	6142	6321	6499	6677	6856	7034	7212	178
71	4	7390	7568	7746	7923	8101	8279	8456	8634	8811	8989	178
89	5	9166	9343	9520	9698	9875	390051	390228	390405	390582	390759	177
106	6	390935	391112	391288	391464	391641	1817	1993	2169	2345	2521	176
124	7	2697	2873	3048	3224	3400	3575	3751	3926	4101	4277	176
142	8	4452	4627	4802	4977	5152	5326	5501	5676	5850	6025	175
159	9	6199	6374	6548	6722	6896	7071	7245	7419	7592	7766	174
	250	397940	398114	398287	398461	398634	398808	398981	399154	399328	399502	173
17	1	9674	9847	400020	400192	400365	400538	400711	400883	401056	401229	173
34	2	401401	401573	1745	1917	2089	2261	2433	2605	2777	2949	172
51	3	3121	3292	3464	3635	3807	3978	4149	4320	4492	4663	171
68	4	4834	5005	5176	5346	5517	5688	5858	6029	6199	6370	171
85	5	6540	6710	6881	7051	7221	7391	7561	7731	7901	8070	170
102	6	8240	8410	8579	8749	8918	9087	9257	9426	9595	9764	169
119	7	9933	410102	410271	410440	410609	410777	410946	411114	411283	411451	169
136	8	417620	1788	1956	2124	2293	2461	2629	2796	2964	3132	168
153	9	3300	3467	3635	3803	3970	4137	4305	4472	4639	4806	167
	260	414973	415140	415307	415474	415641	415808	415974	416141	416308	416474	167
16	1	6641	6807	6973	7139	7306	7472	7638	7804	7970	8135	166
33	2	8301	8467	8633	8798	8964	9129	9295	9460	9625	9791	165
49	3	9956	420121	420286	420451	420616	420781	420945	421110	421275	421439	165
66	4	421604	1768	1933	2097	2261	2426	2590	2754	2918	3082	164
82	5	3246	3410	3574	3737	3901	4065	4228	4392	4555	4718	164
98	6	4882	5045	5208	5371	5534	5697	5860	6023	6186	6349	163
115	7	6511	6674	6836	6999	7161	7324	7486	7648	7811	7973	162
131	8	8135	8297	8459	8621	8783	8944	9106	9268	9429	9591	162
148	9	9752	9914	430075	430236	430398	430559	430720	430881	431042	431203	161
	270	431364	431525	431685	431846	432007	432167	432328	432488	432649	432809	161
16	1	2969	3130	3290	3450	3610	3770	3930	4090	4249	4409	160
32	2	4569	4729	4888	5048	5207	5367	5526	5685	5844	6004	159
47	3	6163	6322	6481	6640	6799	6957	7116	7275	7433	7592	159
63	4	7751	7909	8067	8226	8384	8542	8701	8859	9017	9175	158
79	5	9333	9491	9648	9806	9964	440122	440279	440437	440594	440752	158
95	6	440909	441066	441224	441381	441538	1695	1852	2009	2166	2323	157
111	7	2480	2637	2793	2950	3106	3263	3419	3576	3732	3889	157
126	8	4045	4201	4357	4513	4669	4825	4981	5137	5293	5449	156
142	9	5604	5760	5915	6071	6226	6382	6537	6692	6848	7003	155

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	280	447158	447313	447468	447623	447778	447933	448088	448242	448397	448552	155
15	1	8706	8861	9015	9170	9324	9478	9633	9787	9941	450095	154
31	2	450249	450403	450557	450711	450865	451018	451172	451326	451479	1633	154
46	3	1786	1940	2093	2247	2400	2553	2706	2859	3012	3165	153
61	4	3318	3471	3624	3777	3930	4082	4235	4387	4540	4692	153
77	5	4845	4997	5150	5302	5454	5606	5758	5910	6062	6214	152
92	6	6366	6518	6670	6821	6973	7125	7276	7428	7579	7731	152
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122	8	9392	9543	9694	9845	9995	460146	460296	460447	460597	460748	151
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88	6	471292	471438	1585	1732	1878	2025	2171	2318	2464	2610	146
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43	3	1443	1586	1729	1872	2016	2159	2302	2445	2588	2731	143
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41	3	5544	5683	5822	5960	6099	6238	6376	6515	6653	6791	139
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83	6	9687	9824	9962	500099	500236	500374	500511	500648	500785	500922	137
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26	2	521138	521269	1400	1530	1661	1792	1922	2053	2183	2314	131
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38	3	5294	5421	5547	5674	5800	5927	6053	6180	6306	6432	126
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	360	556303	556423	556544	556664	556785	556905	557026	557146	557267	557387	120
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71	6	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	119
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	370	568202	568319	568436	568554	568671	568788	568905	569023	569140	569257	117
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23	2	570543	570660	570776	570893	571010	571126	1243	1359	1476	1592	117
35	3	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
46	4	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	116
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57	5	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	113
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77	7	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
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32	3	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	108
43	4	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
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96	9	1723	1829	1936	2042	2148	2254	2360	2466	2572	2678	106
	410	612784	612890	612996	613102	613207	613313	613419	613525	613630	613736	106
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21	2	4897	5003	5108	5213	5319	5424	5529	5634	5740	5845	105
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42	4	7000	7105	7210	7315	7420	7525	7629	7734	7839	7943	105
53	5	8048	8153	8257	8362	8466	8571	8676	8780	8884	8989	105
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74	7	620136	620240	620344	620448	620552	620656	620760	620864	620968	1072	104
84	8	1176	1280	1384	1488	1592	1695	1799	1903	2007	2110	104
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31	3	6340	6443	6546	6648	6751	6853	6956	7058	7161	7263	103
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82	8	1444	1545	1647	1748	1849	1951	2052	2153	2255	2356	101
92	9	2457	2559	2660	2761	2862	2963	3064	3165	3266	3367	101
	430	633468	633569	633670	633771	633872	633973	634074	634175	634276	634376	101
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	440	643453	643551	643650	643749	643847	643946	644044	644143	644242	644340	98
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20	2	5422	5521	5619	5717	5815	5913	6011	6110	6208	6306	98
29	3	6404	6502	6600	6698	6796	6894	6992	7089	7187	7285	98
39	4	7383	7481	7579	7676	7774	7872	7969	8067	8165	8262	98
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38	4	7056	7152	7247	7343	7438	7534	7629	7725	7820	7916	96
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58	6	8965	9060	9155	9250	9346	9441	9535	9631	9726	9821	95
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77	8	660865	0960	1055	1150	1245	1339	1434	1529	1623	1718	95
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38	4	6518	6612	6705	6799	6892	6986	7079	7173	7266	7360	94
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85	9	1173	1265	1358	1451	1543	1636	1728	1821	1913	2005	93
	470	672098	672190	672283	672375	672467	672560	672652	672744	672836	672929	92
9	1	3021	3113	3205	3297	3390	3482	3574	3666	3758	3850	92
18	2	3942	4034	4126	4218	4310	4402	4494	4586	4677	4769	92
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37	4	5778	5870	5962	6053	6145	6236	6328	6419	6511	6602	92
46	5	6694	6785	6876	6968	7059	7151	7242	7333	7424	7516	91
55	6	7607	7698	7789	7881	7972	8063	8154	8245	8336	8427	91
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35	4	3727	3815	3903	3991	4078	4166	4254	4342	4430	4517	88
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53	6	5482	5569	5657	5744	5832	5919	6007	6094	6182	6269	87
62	7	6356	6444	6531	6618	6706	6793	6880	6968	7055	7142	87
70	8	7229	7317	7404	7491	7578	7665	7752	7839	7926	8014	87
79	9	8101	8188	8275	8362	8449	8535	8622	8709	8796	8883	87
	500	698970	699057	699144	699231	699317	699404	699491	699578	699664	699751	87
9	1	9838	9924	700011	700098	700184	700271	700358	700444	700531	700617	87
17	2	700704	700790	0877	0963	1050	1136	1222	1309	1395	1482	86
26	3	1568	1654	1741	1827	1913	1999	2086	2172	2258	2344	86
34	4	2431	2517	2603	2689	2775	2861	2947	3033	3119	3205	86
43	5	3291	3377	3463	3549	3635	3721	3807	3895	3979	4065	86
52	6	4151	4236	4322	4408	4494	4579	4665	4751	4837	4922	86
60	7	5008	5094	5179	5265	5350	5436	5522	5607	5693	5778	86
69	8	5864	5949	6035	6120	6206	6291	6376	6462	6547	6632	85
77	9	6718	6803	6888	6974	7059	7144	7229	7315	7400	7485	85
	510	707570	707655	707740	707826	707911	707996	708081	708166	708251	708336	85
8	1	8121	8506	8591	8676	8761	8846	8931	9015	9100	9185	85
17	2	9270	9355	9440	9524	9609	9694	9779	9863	9948	710033	85
25	3	710117	710202	710287	710371	710456	710540	710625	710710	710794	0879	85
34	4	0963	1048	1132	1217	1301	1385	1470	1554	1639	1723	84
42	5	1807	1892	1976	2060	2144	2229	2313	2397	2481	2566	84
50	6	2650	2734	2818	2902	2986	3070	3154	3238	3323	3407	84
59	7	3491	3575	3659	3742	3826	3910	3994	4078	4162	4246	84
67	8	4330	4414	4497	4581	4665	4749	4833	4916	5000	5084	84
76	9	5167	5251	5335	5418	5502	5586	5669	5753	5836	5920	84

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8	1	6838	6921	7004	7088	7171	7254	7338	7421	7504	7587	83
17	2	7671	7754	7837	7920	8003	8086	8169	8253	8336	8419	83
25	3	8502	8585	8668	8751	8834	8917	9000	9083	9165	9248	83
33	4	9331	9414	9497	9580	9663	9745	9828	9911	9994	720077	83
41	5	720159	720242	720325	720407	720490	720573	720655	720738	720821	0903	83
50	6	0986	1068	1151	1233	1316	1398	1481	1563	1646	1728	82
58	7	1811	1893	1975	2058	2140	2222	2305	2387	2469	2552	82
66	8	2634	2716	2798	2881	2963	3045	3127	3209	3291	3374	82
75	9	3456	3538	3620	3702	3784	3866	3948	4030	4112	4194	82
	530	724276	724358	724440	724522	724604	724685	724767	724849	724931	725013	82
8	1	5095	5176	5258	5340	5422	5503	5585	5667	5748	5830	82
16	2	5912	5998	6075	6156	6238	6320	6401	6483	6564	6646	82
24	3	6727	6809	6890	6972	7053	7134	7216	7297	7379	7460	81
32	4	7541	7623	7704	7785	7866	7948	8029	8110	8191	8273	81
41	5	8354	8435	8516	8597	8678	8759	8841	8922	9003	9084	81
49	6	9165	9246	9327	9408	9489	9570	9651	9732	9813	9893	81
57	7	9974	730055	730136	730217	730298	730378	730459	730540	730621	730702	81
65	8	730782	0863	0944	1024	1105	1186	1266	1347	1428	1508	81
73	9	1589	1669	1750	1830	1911	1991	2072	2152	2233	2313	81
	540	732394	732474	732555	732635	732715	732796	732876	732956	733037	733117	80
8	1	3197	3278	3358	3438	3518	3598	3679	3759	3839	3919	80
16	2	3999	4079	4160	4240	4320	4400	4480	4560	4640	4720	80
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40	5	6397	6476	6556	6635	6715	6795	6874	6954	7034	7113	80
48	6	7193	7272	7352	7431	7511	7590	7670	7749	7829	7908	79
56	7	7987	8067	8146	8225	8305	8384	8463	8543	8622	8701	79
64	8	8781	8860	8939	9018	9097	9177	9256	9335	9414	9493	79
72	9	9572	9651	9731	9810	9889	9968	740047	740126	740205	740284	79
	550	740363	740442	740521	740600	740678	740757	740836	740915	740994	741073	79
8	1	1152	1230	1309	1388	1467	1546	1624	1703	1782	1860	79
16	2	1939	2018	2096	2175	2254	2332	2411	2489	2568	2647	79
23	3	2725	2804	2882	2961	3039	3118	3196	3275	3353	3431	78
31	4	3510	3588	3667	3745	3823	3902	3980	4058	4136	4215	78
39	5	4293	4371	4449	4528	4606	4684	4762	4840	4919	4997	78
47	6	5075	5153	5231	5309	5387	5465	5543	5621	5699	5777	78
55	7	5855	5933	6011	6089	6167	6245	6323	6401	6479	6556	78
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	560	748188	748266	748343	748421	748498	748576	748653	748731	748808	748885	77
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15	2	9736	9814	9891	9968	750045	750123	750200	750277	750354	750431	77
23	3	750508	750586	750663	750740	0817	0894	0971	1048	1125	1202	77
31	4	1279	1356	1433	1510	1587	1664	1741	1818	1895	1972	77
39	5	2048	2125	2202	2279	2356	2433	2509	2586	2663	2740	77
46	6	2816	2893	2970	3047	3123	3200	3277	3353	3430	3506	77
54	7	3583	3660	3736	3813	3889	3966	4042	4119	4195	4272	77
62	8	4348	4425	4501	4578	4654	4730	4807	4883	4960	5036	76
69	9	5112	5189	5265	5341	5417	5494	5570	5646	5722	5799	76
	570	755875	755951	756027	756103	756180	756256	756332	756408	756484	756560	76
8	1	6636	6712	6788	6864	6940	7016	7092	7168	7244	7320	76
15	2	7396	7472	7548	7624	7700	7775	7851	7927	8003	8079	76
23	3	8155	8230	8306	8382	8458	8533	8609	8685	8761	8836	76
30	4	8912	8988	9063	9139	9214	9290	9366	9441	9517	9592	76
38	5	9668	9743	9819	9894	9970	760045	760121	760196	760272	760347	75
46	6	760422	760498	760573	760649	760724	0799	0875	0950	1025	1101	75
53	7	1176	1251	1326	1402	1477	1552	1627	1702	1778	1853	75
61	8	1928	2003	2078	2153	2228	2303	2378	2453	2529	2604	75
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	580	763428	763503	763578	763653	763727	763802	763877	763952	764027	764101	75
7	1	4176	4251	4326	4400	4475	4550	4624	4699	4774	4848	75
15	2	4923	4998	5072	5147	5221	5296	5370	5445	5520	5594	75
22	3	5669	5743	5818	5892	5966	6041	6115	6190	6264	6338	74
30	4	6413	6487	6562	6636	6710	6785	6859	6933	7007	7082	74
37	5	7156	7230	7304	7379	7453	7527	7601	7675	7749	7823	74
44	6	7898	7972	8046	8120	8194	8268	8342	8416	8490	8564	74
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59	8	9377	9451	9525	9599	9673	9746	9820	9894	9968	770042	74
67	9	770115	770189	770263	770336	770410	770484	770557	770631	770705	0778	74
	590	770852	770926	770999	771073	771146	771220	771293	771367	771440	771514	74
7	1	1587	1661	1734	1808	1881	1955	2028	2102	2175	2248	73
15	2	2322	2395	2468	2542	2615	2688	2762	2835	2908	2981	73
22	3	3055	3128	3201	3274	3348	3421	3494	3567	3640	3713	73
29	4	3786	3860	3933	4006	4079	4152	4225	4298	4371	4444	73
37	5	4517	4590	4663	4736	4809	4882	4955	5028	5100	5173	73
44	6	5246	5319	5392	5465	5538	5610	5683	5756	5829	5902	73
51	7	5974	6047	6120	6193	6265	6338	6411	6483	6556	6629	73
58	8	6701	6774	6846	6919	6992	7064	7137	7209	7282	7354	73
66	9	7427	7499	7572	7644	7717	7789	7862	7934	8006	8079	72
	600	778151	778224	778296	778368	778441	778513	778585	778658	778730	778802	72
7	1	8874	8947	9019	9091	9163	9236	9308	9380	9452	9524	72
14	2	9596	9668	9741	9813	9885	9957	780029	780101	780173	780245	72
22	3	780317	780389	780461	780533	780605	780677	0749	0821	0893	0965	72
29	4	1037	1109	1181	1253	1324	1396	1468	1540	1612	1684	72
36	5	1755	1827	1899	1971	2042	2114	2186	2258	2329	2401	72
43	6	2473	2544	2616	2688	2759	2831	2902	2974	3046	3117	72
50	7	3189	3260	3332	3403	3475	3546	3618	3689	3761	3832	71
58	8	3904	3975	4046	4118	4189	4261	4332	4403	4475	4546	71
65	9	4617	4689	4760	4831	4902	4974	5045	5116	5187	5259	71
	610	785330	785401	785472	785543	785615	785686	785757	785828	785899	785970	71
7	1	6041	6112	6183	6254	6325	6396	6467	6538	6609	6680	71
14	2	6751	6822	6893	6964	7035	7106	7177	7248	7319	7390	71
21	3	7460	7531	7602	7673	7744	7815	7885	7956	8027	8098	71
28	4	8168	8239	8310	8381	8451	8522	8593	8663	8734	8804	71
36	5	8875	8946	9016	9087	9157	9228	9299	9369	9440	9510	71
43	6	9581	9651	9722	9792	9863	9933	790004	790074	790144	790215	70
50	7	790285	790356	790426	790496	790567	790637	0707	0778	0848	0918	70
57	8	0988	1059	1129	1199	1269	1340	1410	1480	1550	1620	70
64	9	1691	1761	1831	1901	1971	2041	2111	2181	2252	2322	70
	620	792392	792462	792532	792602	792672	792742	792812	792882	792952	793022	70
7	1	3092	3162	3231	3301	3371	3441	3511	3581	3651	3721	70
14	2	3790	3860	3930	4000	4070	4139	4209	4279	4349	4418	70
21	3	4488	4558	4627	4697	4767	4836	4906	4976	5045	5115	70
28	4	5185	5254	5324	5393	5463	5532	5602	5672	5741	5811	70
35	5	5880	5949	6019	6088	6158	6227	6297	6366	6436	6505	69
42	6	6574	6644	6713	6782	6852	6921	6990	7060	7129	7198	69
49	7	7268	7337	7406	7475	7545	7614	7683	7752	7821	7890	69
56	8	7960	8029	8098	8167	8236	8305	8374	8443	8513	8582	69
63	9	8651	8720	8789	8858	8927	8996	9065	9134	9203	9272	69
	630	799341	799409	799478	799547	799616	799685	799754	799823	799892	799961	69
7	1	800029	800098	800167	800236	800305	800373	800442	800511	800580	800648	69
14	2	0717	0786	0854	0923	0992	1061	1129	1198	1266	1335	69
21	3	1404	1472	1541	1609	1678	1747	1815	1884	1952	2021	69
28	4	2089	2158	2226	2295	2363	2432	2500	2568	2637	2705	69
35	5	2774	2842	2910	2979	3047	3116	3184	3252	3321	3389	68
41	6	3457	3525	3594	3662	3730	3798	3867	3935	4003	4071	68
48	7	4139	4208	4276	4344	4412	4480	4548	4616	4685	4753	68
55	8	4821	4889	4957	5025	5093	5161	5229	5297	5365	5433	68
62	9	5501	5569	5637	5705	5773	5841	5908	5976	6044	6112	68

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	640	806180	806248	806316	806384	806451	806519	806587	806655	806723	806790	68
7	1	6858	6926	6994	7061	7129	7197	7264	7332	7400	7467	68
13	2	7535	7603	7670	7738	7806	7873	7941	8008	8076	8143	68
20	3	8211	8279	8346	8414	8481	8549	8616	8684	8751	8818	67
27	4	8886	8953	9021	9088	9156	9223	9290	9358	9425	9492	67
34	5	9560	9627	9694	9762	9829	9896	9964	810031	810098	810165	67
40	6	810233	810300	810367	810434	810501	810569	810636	0703	0770	0837	67
47	7	0904	0971	1039	1106	1173	1240	1307	1374	1441	1508	67
54	8	1575	1642	1709	1776	1843	1910	1977	2044	2111	2178	67
60	9	2245	2312	2379	2445	2512	2579	2646	2713	2780	2847	67
	650	812913	812980	813047	813114	813181	813247	813314	813381	813448	813514	67
7	1	3581	3648	3714	3781	3848	3914	3981	4048	4114	4181	67
13	2	4248	4314	4381	4447	4514	4581	4647	4714	4780	4847	67
20	3	4913	4980	5046	5113	5179	5246	5312	5378	5445	5511	66
26	4	5578	5644	5711	5777	5843	5910	5976	6042	6109	6175	66
33	5	6241	6308	6374	6440	6506	6573	6639	6705	6771	6838	66
40	6	6904	6970	7036	7102	7169	7235	7301	7367	7433	7499	66
46	7	7565	7631	7698	7764	7830	7896	7962	8028	8094	8160	66
53	8	8226	8292	8358	8424	8490	8556	8622	8688	8754	8820	66
59	9	8885	8951	9017	9083	9149	9215	9281	9346	9412	9478	66
	660	819544	819610	819676	819741	819807	819873	819939	820004	820070	820136	66
7	1	820201	820267	820333	820399	820464	820530	820595	0661	0727	0792	66
13	2	0858	0924	0989	1055	1120	1186	1251	1317	1382	1448	66
20	3	1514	1579	1645	1710	1775	1841	1906	1972	2037	2103	65
26	4	2168	2233	2299	2364	2430	2495	2560	2626	2691	2756	65
33	5	2822	2887	2952	3018	3083	3148	3213	3279	3344	3409	65
39	6	3474	3539	3605	3670	3735	3800	3865	3930	3996	4061	65
46	7	4126	4191	4256	4321	4386	4451	4516	4581	4646	4711	65
52	8	4776	4841	4906	4971	5036	5101	5166	5231	5296	5361	65
59	9	5426	5491	5556	5621	5686	5751	5815	5880	5945	6010	65
	670	826075	826140	826204	826269	826334	826399	826464	826528	826593	826658	65
6	1	6723	6787	6852	6917	6981	7046	7111	7175	7240	7305	65
13	2	7369	7434	7499	7563	7628	7692	7757	7821	7886	7951	65
19	3	8015	8080	8144	8209	8273	8338	8402	8467	8531	8595	64
26	4	8660	8724	8789	8853	8918	8982	9046	9111	9175	9239	64
32	5	9304	9368	9432	9497	9561	9625	9690	9754	9818	9882	64
38	6	9947	830011	830075	830139	830204	830268	830332	830396	830460	830525	64
45	7	830589	0653	0717	0781	0845	0909	0973	1037	1102	1166	64
51	8	1230	1294	1358	1422	1486	1550	1614	1678	1742	1806	64
58	9	1870	1934	1998	2062	2126	2189	2253	2317	2381	2445	64
	680	832509	832573	832637	832700	832764	832828	832892	832956	833020	833083	64
6	1	3147	3211	3275	3338	3402	3466	3530	3593	3657	3721	64
13	2	3784	3848	3912	3975	4039	4103	4166	4230	4294	4357	64
19	3	4421	4484	4548	4611	4675	4739	4802	4866	4929	4993	64
25	4	5056	5120	5183	5247	5310	5373	5437	5500	5564	5627	63
32	5	5691	5754	5817	5881	5944	6007	6071	6134	6197	6261	63
38	6	6324	6387	6451	6514	6577	6641	6704	6767	6830	6894	63
44	7	6957	7020	7083	7146	7210	7273	7336	7399	7462	7525	63
50	8	7588	7652	7715	7778	7841	7904	7967	8030	8093	8156	63
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	690	838849	838912	838975	839038	839101	839164	839227	839289	839352	839415	63
6	1	9478	9541	9604	9667	9729	9792	9855	9918	9981	840043	63
13	2	840106	840169	840232	840294	840357	840420	840482	840545	840608	0671	63
19	3	0733	0796	0859	0921	0984	1046	1109	1172	1234	1297	63
25	4	1359	1422	1485	1547	1610	1672	1735	1797	1860	1922	63
32	5	1985	2047	2110	2172	2235	2297	2360	2422	2484	2547	62
38	6	2609	2672	2734	2796	2859	2921	2983	3046	3108	3170	62
44	7	3233	3295	3357	3420	3482	3544	3606	3669	3731	3793	62
50	8	3855	3918	3980	4042	4104	4166	4229	4291	4353	4415	62
57	9	4477	4539	4601	4664	4726	4788	4850	4912	4974	5036	62

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6	1	5718	5780	5842	5904	5966	6028	6090	6151	6213	6275	62
12	2	6337	6399	6461	6523	6585	6646	6708	6770	6832	6894	62
19	3	6955	7017	7079	7141	7202	7264	7326	7388	7449	7511	62
25	4	7573	7634	7696	7758	7819	7881	7943	8004	8066	8128	62
31	5	8189	8251	8312	8374	8435	8497	8559	8620	8682	8743	62
37	6	8805	8866	8928	8989	9051	9112	9174	9235	9297	9358	61
43	7	9419	9481	9542	9604	9665	9726	9788	9849	9911	9972	61
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56	9	0646	0707	0769	0830	0891	0952	1014	1075	1136	1197	61
	710	851258	851320	851381	851442	851503	851564	851625	851686	851747	851809	61
6	1	1870	1931	1992	2053	2114	2175	2236	2297	2358	2419	61
12	2	2480	2541	2602	2663	2724	2785	2846	2907	2968	3029	61
18	3	3090	3150	3211	3272	3333	3394	3455	3516	3577	3637	61
24	4	3698	3759	3820	3881	3941	4002	4063	4124	4185	4245	61
31	5	4306	4367	4428	4488	4549	4610	4670	4731	4792	4852	61
37	6	4913	4974	5034	5095	5156	5216	5277	5337	5398	5459	61
43	7	5519	5580	5640	5701	5761	5822	5882	5943	6003	6064	61
49	8	6124	6185	6245	6306	6366	6427	6487	6548	6608	6668	60
55	9	6729	6789	6850	6910	6970	7031	7091	7152	7212	7272	60
	720	857332	857393	857453	857513	857574	857634	857694	857755	857815	857875	60
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12	2	8537	8597	8657	8718	8778	8838	8898	8958	9018	9078	60
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	730	863323	863382	863442	863501	863561	863620	863680	863739	863799	863858	59
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12	2	4511	4570	4630	4689	4748	4808	4867	4926	4985	5045	59
18	3	5104	5163	5222	5282	5341	5400	5459	5519	5578	5637	59
24	4	5696	5755	5814	5874	5933	5992	6051	6110	6169	6228	59
30	5	6287	6346	6405	6465	6524	6583	6642	6701	6760	6819	59
35	6	6878	6937	6996	7055	7114	7173	7232	7291	7350	7409	59
41	7	7467	7526	7585	7644	7703	7762	7821	7880	7939	7998	59
47	8	8056	8115	8174	8233	8292	8350	8409	8468	8527	8586	59
53	9	8644	8703	8762	8821	8879	8938	8997	9056	9114	9173	59
	740	869232	869290	869349	869408	869466	869525	869584	869642	869701	869760	59
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17	3	0989	1047	1106	1164	1223	1281	1339	1398	1456	1515	58
23	4	1573	1631	1690	1748	1806	1865	1923	1981	2040	2098	58
29	5	2156	2215	2273	2331	2389	2448	2506	2564	2622	2681	58
35	6	2739	2797	2855	2913	2972	3030	3088	3146	3204	3262	58
41	7	3321	3379	3437	3495	3553	3611	3669	3727	3785	3844	58
46	8	3902	3960	4018	4076	4134	4192	4250	4308	4366	4424	58
52	9	4482	4540	4598	4656	4714	4772	4830	4888	4945	5003	58
	750	875061	875119	875177	875235	875293	875351	875409	875466	875524	875582	58
6	1	5640	5698	5756	5813	5871	5929	5987	6045	6102	6160	58
12	2	6218	6276	6333	6391	6449	6507	6564	6622	6680	6737	58
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23	4	7371	7429	7487	7544	7602	7659	7717	7774	7832	7889	58
29	5	7947	8004	8062	8119	8177	8234	8292	8349	8407	8464	57
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41	7	9096	9153	9211	9268	9325	9383	9440	9497	9555	9612	57
46	8	9669	9726	9784	9841	9898	9956	880013	880070	880127	880185	57
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11	2	1955	2012	2069	2126	2183	2240	2297	2354	2411	2468	57
17	3	2525	2581	2638	2695	2752	2809	2866	2923	2980	3037	57
23	4	3093	3150	3207	3264	3321	3377	3434	3491	3548	3605	57
29	5	3661	3718	3775	3832	3888	3945	4002	4059	4115	4172	57
34	6	4229	4285	4342	4399	4455	4512	4569	4625	4682	4739	57
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46	8	5361	5418	5474	5531	5587	5644	5700	5757	5813	5870	57
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17	3	8179	8236	8292	8348	8404	8460	8516	8573	8629	8685	56
22	4	8741	8797	8853	8909	8965	9021	9077	9134	9190	9246	56
28	5	9302	9358	9414	9470	9526	9582	9638	9694	9750	9806	56
34	6	9862	9918	9974	890030	890086	890141	890197	890253	890309	890365	56
39	7	890421	890477	890533	0589	0645	0700	0756	0812	0868	0924	56
45	8	0980	1035	1091	1147	1203	1259	1314	1370	1426	1482	56
50	9	1537	1593	1649	1705	1760	1816	1872	1928	1983	2039	56
	780	892095	892150	892206	892262	892317	892373	892429	892484	892540	892595	56
6	1	2651	2707	2762	2818	2873	2929	2985	3040	3096	3151	56
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17	3	3762	3817	3873	3928	3984	4039	4094	4150	4205	4261	55
22	4	4316	4371	4427	4482	4538	4593	4648	4704	4759	4814	55
27	5	4870	4925	4980	5036	5091	5146	5201	5257	5312	5367	55
33	6	5423	5478	5533	5588	5644	5699	5754	5809	5864	5920	55
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	790	897627	897682	897737	897792	897847	897902	897957	898012	898067	898122	55
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17	3	9273	9328	9383	9437	9492	9547	9602	9656	9711	9766	55
22	4	9821	9875	9930	9985	900039	900094	900149	900203	900258	900312	55
27	5	900367	900422	900476	900531	0586	0640	0695	0749	0804	0859	55
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38	7	1458	1513	1567	1622	1676	1731	1785	1840	1894	1948	54
44	8	2003	2057	2112	2166	2221	2275	2329	2384	2438	2492	54
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22	4	5256	5310	5364	5418	5472	5526	5580	5634	5688	5742	54
27	5	5796	5850	5904	5958	6012	6066	6119	6173	6227	6281	54
32	6	6335	6389	6443	6497	6551	6604	6658	6712	6766	6820	54
38	7	6874	6927	6981	7035	7089	7143	7196	7250	7304	7358	54
43	8	7411	7465	7519	7573	7626	7680	7734	7787	7841	7895	54
49	9	7949	8002	8056	8110	8163	8217	8270	8324	8378	8431	54
	810	908485	908539	908592	908646	908699	908753	908807	908860	908914	908967	54
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16	3	910091	910144	910197	910251	910304	910358	910411	910464	910518	0571	53
21	4	0624	0678	0731	0784	0838	0891	0944	0998	1051	1104	53
27	5	1158	1211	1264	1317	1371	1424	1477	1530	1584	1637	53
32	6	1690	1743	1797	1850	1903	1956	2009	2063	2116	2169	53
37	7	2222	2275	2328	2381	2435	2488	2541	2594	2647	2700	53
42	8	2753	2806	2859	2913	2966	3019	3072	3125	3178	3231	53
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5	1	4343	4396	4449	4502	4555	4608	4660	4713	4766	4819	53
11	2	4872	4925	4977	5030	5083	5136	5189	5241	5294	5347	53
16	3	5400	5453	5505	5558	5611	5664	5716	5769	5822	5875	53
21	4	5927	5980	6033	6085	6138	6191	6243	6296	6349	6401	53
27	5	6454	6507	6559	6612	6664	6717	6770	6822	6875	6927	53
32	6	6980	7033	7085	7138	7190	7243	7295	7348	7400	7453	53
37	7	7506	7558	7611	7663	7716	7768	7820	7873	7925	7978	52
42	8	8030	8083	8135	8188	8240	8293	8345	8397	8450	8502	52
48	9	8555	8607	8659	8712	8764	8816	8869	8921	8973	9026	52
	830	919078	919130	919183	919235	919287	919340	919392	919444	919496	919549	52
5	1	9601	9653	9706	9758	9810	9862	9914	9967	920019	920071	52
10	2	920123	920176	920228	920280	920332	920384	920436	920489	0541	0593	52
16	3	0645	0697	0749	0801	0853	0906	0958	1010	1062	1114	52
21	4	1166	1218	1270	1322	1374	1426	1478	1530	1582	1634	52
26	5	1686	1738	1790	1842	1894	1946	1998	2050	2102	2154	52
31	6	2206	2258	2310	2362	2414	2466	2518	2570	2622	2674	52
36	7	2725	2777	2829	2881	2933	2985	3037	3089	3140	3192	52
42	8	3244	3296	3348	3399	3451	3503	3555	3607	3658	3710	52
47	9	3762	3814	3865	3917	3969	4021	4072	4124	4176	4228	52
	840	924279	924331	924383	924434	924486	924538	924589	924641	924693	924744	52
5	1	4796	4848	4899	4951	5003	5054	5106	5157	5209	5261	52
10	2	5312	5364	5415	5467	5518	5570	5621	5673	5725	5776	52
15	3	5828	5879	5931	5982	6034	6085	6137	6188	6240	6291	51
20	4	6342	6394	6445	6497	6548	6600	6651	6702	6754	6805	51
26	5	6857	6908	6959	7011	7062	7114	7165	7216	7268	7319	51
31	6	7370	7422	7473	7524	7576	7627	7678	7730	7781	7832	51
36	7	7883	7935	7986	8037	8088	8140	8191	8242	8293	8345	51
41	8	8396	8447	8498	8549	8601	8652	8703	8754	8805	8857	51
46	9	8908	8959	9010	9061	9112	9163	9215	9266	9317	9368	51
	850	929419	929470	929521	929572	929623	929674	929725	929776	929827	929879	51
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10	2	930440	930491	0542	0592	0643	0694	0745	0796	0847	0898	51
15	3	0949	1000	1051	1102	1153	1204	1254	1305	1356	1407	51
20	4	1458	1509	1560	1610	1661	1712	1763	1814	1865	1915	51
26	5	1966	2017	2068	2118	2169	2220	2271	2322	2372	2423	51
31	6	2474	2524	2575	2626	2677	2727	2778	2829	2879	2930	51
36	7	2981	3031	3082	3133	3183	3234	3285	3335	3386	3437	51
41	8	3487	3538	3589	3639	3690	3740	3791	3841	3892	3943	51
46	9	3993	4044	4094	4145	4195	4246	4296	4347	4397	4448	51
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10	2	5507	5558	5608	5658	5709	5759	5809	5860	5910	5960	50
15	3	6011	6061	6111	6162	6212	6262	6313	6363	6413	6463	50
20	4	6514	6564	6614	6665	6715	6765	6815	6865	6916	6966	50
25	5	7016	7066	7117	7167	7217	7267	7317	7367	7418	7468	50
30	6	7518	7568	7618	7668	7718	7769	7819	7869	7919	7969	50
35	7	8019	8069	8119	8169	8219	8269	8320	8370	8420	8470	50
40	8	8520	8570	8620	8670	8720	8770	8820	8870	8920	8970	50
45	9	9020	9070	9120	9170	9220	9270	9320	9369	9419	9469	50
	870	939519	939569	939619	939669	939719	939769	939819	939869	939918	939968	50
5	1	940018	940068	940118	940168	940218	940267	940317	940367	940417	940467	50
10	2	0516	0566	0616	0666	0716	0765	0815	0865	0915	0964	50
15	3	1014	1064	1114	1163	1213	1263	1313	1362	1412	1462	50
20	4	1511	1561	1611	1660	1710	1760	1809	1859	1909	1958	50
25	5	2008	2058	2107	2157	2207	2256	2306	2355	2405	2455	50
30	6	2504	2554	2603	2653	2702	2752	2801	2851	2901	2950	50
35	7	3000	3049	3099	3148	3198	3247	3297	3346	3396	3445	49
40	8	3495	3544	3593	3643	3692	3742	3792	3841	3890	3939	49
45	9	3989	4038	4088	4137	4186	4236	4285	4335	4384	4433	49

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5	1	4976	5025	5074	5124	5173	5222	5272	5321	5370	5419	49
10	2	5469	5518	5567	5616	5665	5715	5764	5813	5862	5912	49
15	3	5961	6010	6059	6108	6157	6207	6256	6305	6354	6403	49
20	4	6452	6501	6551	6600	6649	6698	6747	6796	6845	6894	49
25	5	6943	6992	7041	7090	7140	7189	7238	7287	7336	7385	49
29	6	7434	7483	7532	7581	7630	7679	7728	7777	7826	7875	49
34	7	7924	7973	8022	8070	8119	8168	8217	8266	8315	8364	49
39	8	8413	8462	8511	8560	8609	8657	8706	8755	8804	8853	49
44	9	8902	8951	8999	9048	9097	9146	9195	9244	9292	9341	49
	890	949390	949439	949488	949536	949585	949634	949683	949731	949780	949829	49
5	1	9878	9926	9975	950024	950073	950121	950170	950219	950267	950316	49
10	2	950365	950414	950462	0511	0560	0608	0657	0706	0754	0803	49
15	3	0851	0900	0949	0997	1046	1095	1143	1192	1240	1289	49
20	4	1338	1386	1435	1483	1532	1580	1629	1677	1726	1775	49
24	5	1823	1872	1920	1969	2017	2066	2114	2163	2211	2260	48
29	6	2308	2356	2405	2453	2502	2550	2599	2647	2696	2744	48
34	7	2792	2841	2889	2938	2986	3034	3083	3131	3180	3228	48
39	8	3276	3325	3373	3421	3470	3518	3566	3615	3663	3711	48
44	9	3760	3808	3856	3905	3953	4001	4049	4098	4146	4194	48
	900	954243	954291	954339	954387	954435	954484	954532	954580	954628	954677	48
5	1	4725	4773	4821	4869	4918	4966	5014	5062	5110	5158	48
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14	3	5688	5736	5784	5832	5880	5928	5976	6024	6072	6120	48
19	4	6168	6216	6265	6313	6361	6409	6457	6505	6553	6601	48
24	5	6649	6697	6745	6793	6840	6888	6936	6984	7032	7080	48
29	6	7128	7176	7224	7272	7320	7368	7416	7464	7512	7559	48
34	7	7607	7655	7703	7751	7799	7847	7894	7942	7990	8038	48
38	8	8086	8134	8181	8229	8277	8325	8373	8421	8468	8516	48
43	9	8564	8612	8659	8707	8755	8803	8850	8898	8946	8994	48
	910	959041	959089	959137	959185	959232	959280	959328	959375	959423	959471	48
5	1	9518	9566	9614	9661	9709	9757	9804	9852	9900	9947	48
9	2	9995	960042	960090	960138	960185	960233	960281	960328	960376	960423	48
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19	4	0946	0994	1041	1089	1136	1184	1231	1279	1326	1374	47
24	5	1421	1469	1516	1563	1611	1658	1706	1753	1801	1848	47
28	6	1895	1943	1990	2038	2085	2132	2180	2227	2275	2322	47
33	7	2369	2417	2464	2511	2559	2606	2653	2701	2748	2795	47
38	8	2843	2890	2937	2985	3032	3079	3126	3174	3221	3268	47
42	9	3316	3363	3410	3457	3504	3552	3599	3646	3693	3741	47
	920	963788	963835	963882	963929	963977	964024	964071	964118	964165	964212	47
5	1	4260	4307	4354	4401	4448	4495	4542	4590	4637	4684	47
9	2	4731	4778	4825	4872	4919	4966	5013	5061	5108	5155	47
14	3	5202	5249	5296	5343	5390	5437	5484	5531	5578	5625	47
19	4	5672	5719	5766	5813	5860	5907	5954	6001	6048	6095	47
23	5	6142	6189	6236	6283	6329	6376	6423	6470	6517	6564	47
28	6	6611	6658	6705	6752	6799	6845	6892	6939	6986	7033	47
33	7	7080	7127	7173	7220	7267	7314	7361	7408	7454	7501	47
38	8	7548	7595	7642	7688	7735	7782	7829	7875	7922	7969	47
42	9	8016	8062	8109	8156	8203	8249	8296	8343	8390	8436	47
	930	968483	968530	968576	968623	968670	968716	968763	968810	968856	968903	47
5	1	8950	8996	9043	9090	9136	9183	9229	9276	9323	9369	47
9	2	9416	9463	9509	9556	9602	9649	9695	9742	9789	9835	47
14	3	9882	9928	9975	970021	970068	970114	970161	970207	970254	970300	47
18	4	970347	970393	970440	0486	0533	0579	0626	0672	0719	0765	46
23	5	0812	0858	0904	0951	0997	1044	1090	1137	1183	1229	46
28	6	1276	1322	1369	1415	1461	1508	1554	1601	1647	1693	46
32	7	1740	1786	1832	1879	1925	1971	2018	2064	2110	2157	46
37	8	2203	2249	2295	2342	2388	2434	2481	2527	2573	2619	46
41	9	2666	2712	2758	2804	2851	2897	2943	2989	3035	3082	46

PP	N.	0	1	2	3	4	5	6	7	8	9	D.
	940	973128	973174	973220	973266	973313	973359	973405	973451	973497	973543	46
5	1	3590	3636	3682	3728	3774	3820	3866	3913	3959	4005	46
9	2	4051	4097	4143	4189	4235	4281	4327	4374	4420	4466	46
14	3	4512	4558	4604	4650	4696	4742	4788	4834	4880	4926	46
18	4	4972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
23	5	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	46
28	6	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	46
32	7	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	46
37	8	6808	6854	6902	6946	6992	7037	7083	7129	7175	7220	46
41	9	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	46
	950	977724	977769	977815	977861	977906	977952	977998	978043	978089	978135	46
5	1	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	46
9	2	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	46
14	3	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	46
18	4	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	46
23	5	980003	980049	980094	980140	980185	980231	980276	980322	980367	980412	45
27	6	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	45
32	7	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	45
36	8	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	45
41	9	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	45
	960	982271	982316	982362	982407	982452	982497	982543	982588	982633	982678	45
5	1	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	45
9	2	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	45
14	3	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	45
18	4	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	45
23	5	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	45
27	6	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	45
32	7	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	45
36	8	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	45
41	9	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	45
	970	986772	986817	986861	986906	986951	986996	987040	987085	987130	987175	45
5	1	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	45
9	2	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	45
14	3	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	45
18	4	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	45
23	5	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	45
27	6	9450	9494	9539	9583	9628	9672	9717	9761	9806	9850	44
32	7	9895	9939	9983	990028	990072	990117	990161	990206	990250	990294	44
36	8	990339	990383	990428	0472	0516	0561	0605	0650	0694	0738	44
41	9	0783	0827	0871	0916	0960	1004	1049	1093	1137	1182	44
	980	991226	991270	991315	991359	991403	991448	991492	991536	991580	991625	44
4	1	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	44
9	2	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	44
13	3	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	44
18	4	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	44
22	5	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	44
26	6	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	44
31	7	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	44
35	8	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	44
40	9	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	44
	990	995635	995679	995723	995767	995811	995854	995898	995942	995986	996030	44
4	1	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	44
9	2	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	44
13	3	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	44
18	4	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	44
22	5	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	44
26	6	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	44
31	7	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	44
35	8	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	44
40	9	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

ANSWERS.

I. Page 2.

1. 1176. 2. 45045. 3. 1140. 4. 1680.
5. 1440. 6. 420. 7. 68640. 8. 58080.

9. 33, 198. 10. 28, 280. 11. 17, 204. 12. 9, 405.
13. 120, 5040. 14. 37, 518. 15. 41, 1435. 16. 46, 2070.
17. 15, 900. 18. 13, 260. 19. 12, 1080. 20. 19, 114.

21. 2016. 22. 2347. 23. 147. 24. 213.
25. 51. 26. 11. 27. 9. 28. 3.

29. $\frac{9}{7}$. 30. $\frac{11}{17}$. 31. $\frac{7}{15}$. 32. $\frac{11}{15}$. 33. $\frac{2}{3}$. 34. $\frac{4}{5}$.
35. $\frac{1}{6}$. 36. $\frac{9}{10}$. 37. $\frac{19}{23}$. 38. $\frac{1}{20}$. 39. $\frac{7}{156}$. 40. $\frac{7}{6}$.
41. $\frac{109}{209}$. 42. $\frac{3}{10}$. 43. $\frac{13}{47}$. 44. $\frac{71}{94}$. 45. $\frac{37}{41}$. 46. $\frac{21}{40}$.
47. $\frac{15}{17}$. 48. $\frac{159}{185}$. 49. $\frac{111}{113}$. 50. $\frac{1}{2}$. 51. $\frac{23}{134}$. 52. $\frac{233}{317}$.
53. $\frac{13}{20}$. 54. $\frac{1}{5}$. 55. $\frac{7}{9}$. 56. $\frac{15}{16}$. 57. $\frac{301}{603}$. 58. $\frac{1}{715}$.
59. $\frac{3}{8}$. 60. $\frac{71}{77}$. 61. $\frac{53}{84}$. 62. $\frac{119}{111}$. 63. $\frac{47320}{50063}$. 64. $\frac{37}{105}$.
65. $\frac{171}{211}$. 66. $\frac{1}{61}$. 67. $\frac{923}{8100}$. 68. $\frac{101}{111}$. 69. $\frac{37}{270}$. 70. $\frac{302}{303}$.

III. Pages 4, 5.

1. $3\frac{88}{315}$. 2. $31\frac{217}{1980}$. 3. $28\frac{59}{168}$. 4. $301\frac{34}{285}$. 5. $633\frac{283}{360}$.
6. $180\frac{883}{720}$. 7. $1\frac{8}{17}$. 8. $\frac{284}{515}$. 9. $31\frac{9}{13}$. 10. $4\frac{5}{96}$.
11. $3\frac{29}{54}$. 12. $4\frac{113}{150}$. 13. $57\frac{1}{3}$. 14. $2\frac{85}{88}$. 15. $1187\frac{1}{6}$.
16. $\frac{17}{28}$. 17. $1\frac{29}{60}$. 18. $2\frac{7}{30}$. 19. $\frac{5}{8}$. 20. $1\frac{73}{144}$.
21. $\frac{11}{36}$. 22. $1\frac{37}{100}$. 23. $9\frac{47}{60}$. 24. $4\frac{115}{252}$. 25. $3\frac{1}{8}$.
26. $6\frac{33}{63}$. 27. $7\frac{7}{120}$. 28. $\frac{23}{60}$. 29. $3\frac{1}{5}$. 30. $\frac{21}{80}$.
31. $\frac{3}{32}$. 32. $\frac{9355}{10374}$. 33. 1200. 34. 14. 35. $6\frac{26}{49}$.
36. $\frac{1}{6}$. 37. $5\frac{5}{8}$. 38. $1\frac{37}{45}$. 39. $2654\frac{7}{16}$. 40. $3\frac{33}{88}$.
41. $6\frac{311}{884}$. 42. $1\frac{11}{45}$. 43. $16\frac{7}{8}$. 44. $1\frac{1}{5}$. 45. $\frac{841}{864}$.
46. $\frac{1}{8}$. 47. $\frac{101}{150}$. 48. $26\frac{1}{4}$. 49. $5\frac{31}{33}$. 50. $4\frac{2}{5}$.
51. $5\frac{1}{10}$. 52. $1\frac{1733}{2520}$. 53. $4\frac{13}{88}$. 54. $\frac{369}{455}$. 55. $1\frac{7551}{11065}$.
56. $\frac{11}{100}$. 57. 10. 58. $5\frac{5}{6}$. 59. $\frac{15}{16}$. 60. $\frac{1}{5}$.
61. $2\frac{1}{7}$. 62. $\frac{5}{50}$. 63. $1\frac{1}{114}$. 64. $\frac{1}{48}$. 65. $193\frac{1}{3}$.
66. $33\frac{3}{8}$. 67. $\frac{1833}{7200}$. 68. $1\frac{1}{10}$. 69. $\frac{1}{6}$. 70. $\frac{5}{756}$.
71. $\frac{2}{8}$. 72. $5\frac{7}{18}$. 73. $\frac{43}{45}$. 74. $\frac{64}{105}$. 75. $\frac{29}{375}$.
76. $\frac{5110}{10000}$. 77. 20. 78. $\frac{12}{65}$. 79. $\frac{76}{185}$. 80. $31\frac{5}{23}$.
81. $\frac{5}{91}$. 82. $6\frac{2}{11}$. 83. $\frac{10753}{12800}$. 84. $\frac{30}{53}$. 85. $14\frac{7}{24}$.
86. $\frac{7}{8}$. 87. $\frac{7}{32}$. 88. $20\frac{13}{25}$. 89. $1\frac{11}{21}$. 90. $8\frac{16}{25}$.
91. $\frac{4}{5}$. 92. 18. 93. $46\frac{2}{41}$.

IV. Page 6.

1. $3\frac{7}{11}$. 2. $1\frac{50}{63}$. 3. 14. 4. $\frac{17}{266}$. 5. $7\frac{18}{19}$.
6. $2\frac{2}{3}$. 7. $1\frac{1}{3}$. 8. $\frac{82}{165}$. 9. $\frac{7}{10}$. 10. $1\frac{1}{2}$.
11. $\frac{1}{2}$. 12. $1\frac{1}{4}$. 13. $12\frac{4}{25}$. 14. $1\frac{13}{35}$. 15. 1.
16. $19\frac{283}{413}$. 17. $4\frac{149}{407}$. 18. $\frac{25}{144}$. 19. $12\frac{3}{11}$. 20. $\frac{5}{1128}$.
21. $\frac{675}{1472}$. 22. $3\frac{3}{5}$. 23. $1\frac{7}{9}$. 24. 28. 25. $\frac{1}{7}$. 26. 1.

V. Pages 7, 8.

1. 36 feet. 2. 880. 3. 48. 4. \$540. 5. $\frac{54}{101}$. 6. $27\frac{8}{21}$.
7. $11\frac{161}{180}$. 8. \$360. 9. $\$32\frac{9}{13}$. 10. $6\frac{1}{11}$. 11. $\frac{1}{1632}$. 12. $1\frac{19}{32}$.
13. $2\frac{7}{8}$. 14. $279\frac{1}{4}$. 15. $\frac{13}{648}$. 16. £1. 18s. 17. $\frac{44}{225}$.
18. \$1371. 19. \$225. 20. $43\frac{9}{16}$. 21. $1\frac{5}{8}$. 22. $35\frac{3}{32}$.
23. $6\frac{248}{620}$. 24. The latter by $\frac{131}{288}$. 25. The former by $1\frac{23}{1056}$.
26. 5. 27. $\frac{349}{35573}$. 28. $\frac{23}{48}$. 29. $90\frac{1147}{2304}$. 30. $3\frac{257}{408}$; $1\frac{283}{408}$; $2\frac{159}{751}$.
31. $\$110\frac{5}{11}$. 32. Increased by $\frac{3}{40}$.

VII. Page 10.

1. $\cdot\dot{3}$. 2. $\cdot\dot{4}$. 3. $\cdot 58\dot{3}$. 4. $\cdot 3\dot{6}$. 5. $\cdot 1\dot{3}$. 6. $\cdot\dot{7}14285\dot{7}$. 7. $\cdot\dot{1}$.
8. $\cdot 0\dot{1}$. 9. $\cdot 00\dot{1}$. 10. $\cdot 0\dot{1}$. 11. $\cdot\dot{0}00\dot{1}$. 12. $\cdot 02\dot{3}$.
13. $5\cdot 230769\dot{}$. 14. $7\cdot 3\dot{8}$. 15. $6\cdot 708\dot{3}$. 16. $2\cdot 0\dot{3}7$. 17. $1\cdot 68\dot{1}$.
18. $20\cdot 13\dot{8}$. 19. $\cdot 10\dot{8}$. 20. $4\cdot\dot{3}76068\dot{}$. 21. $3\cdot\dot{4}$. 22. $1\cdot 0\dot{6}$.
23. $26\cdot 0\dot{7}\dot{4}$. 24. $8\cdot\dot{2}\dot{9}$. 25. $1\cdot\dot{0}099\dot{}$. 26. $\cdot 315476190\dot{}$.
27. $6\cdot 0571428\dot{}$. 28. $\cdot 504\dot{5}$. 29. $\cdot 295138\dot{}$. 30. $\cdot 6\dot{7}\dot{2}$.

51. $3\frac{808}{990}$. 52. $3\frac{391}{495}$. 53. $3\frac{137}{225}$. 54. $10\frac{13}{22}$. 55. $3\frac{1}{18}$.
56. $5\frac{5}{13}$. 57. $4\frac{2}{7}$. 58. $\frac{811889}{999999}$. 59. $\frac{1}{702}$. 60. $11\frac{5}{72}$.
61. $20\frac{17}{24}$. 62. $\frac{11}{1080}$. 63. $\frac{901}{9999}$. 64. $21\frac{1}{333}$. 65. $5\frac{208}{495}$.
66. $17\frac{7}{100}$. 67. $6\frac{879}{990}$. 68. $8\frac{45}{999}$. 69. $17\frac{19}{99}$. 70. $2\frac{1}{180}$.

VIII. Pages 10, 11.

1. 15·44049. 2. 13·38482. 3. 18·28360. 4. ·02020.
5. ·54586. 6. ·02302. 7. 16·41805. 8. 60·49949.
9. ·23036. 10. ·52468. 11. ·01952. 12. ·00652.
13. ·04594. 14. ·03948. 15. ·25793. 16. 56·31427.
17. ·00041. 18. ·00298. 19. 5·15094. 20. 34·5.
21. ·46359. 22. 7·03411. 23. ·03293. 24. 2·88184.
25. ·03911. 26. 3·44424.

VIII. Pages 10, 11 (*continued*).

27. $1\cdot\dot{6}54108\dot{}$. **28.** $1\cdot6302211\dot{}$. **29.** $\cdot8502588315\dot{}$. **30.** $4\cdot94332696\dot{}$.
31. $2\cdot738437\dot{}$. **32.** $1\cdot\dot{7}$. **33.** $1\cdot5\dot{7}$. **34.** $2\cdot36\dot{5}$. **35.** $\cdot6443\dot{1}$.
36. $6\cdot5234416\dot{}$. **37.** $10\cdot756$. **38.** $391\cdot0284\dot{}$. **39.** 24. **40.** 15.
41. 42. **42.** $3\cdot8601\dot{1}$. **43.** $\cdot8\dot{3}$. **44.** $2\cdot\dot{5}$. **45.** $2\cdot\dot{8}$. **46.** $\cdot1821\dot{3}$.

IX. Pages 11, 12.

1. £7. 16s. 8d. **2.** 2 dys. 3 hrs. 20 min. 38·4 sec. **3.** 256 yds. 2·64 in.
4. 4 mi. 800 yds. **5.** 5 min. **6.** 42 wks. 10 hrs. 30 min. **7.** 280°.
8. 1 ft. **9.** 15 sq. yds. 2 ft. $80\frac{16}{25}$ in. **10.** 9 lbs 8 oz. 13 dwt. 8 grs.
11. 3 tons 18 cwt. 36·6 lbs. **12.** 2 sq. yds. 8 ft. 12·78 in.
13. 5 yds. 2 ft. 11·892 in. **14.** $243\frac{3}{4}$ rods. **15.** 17 hrs. 52 min. 39·36 sec. **16.** 58 mi. 303·52 rods. **17.** £3. 13s. 2d.
18. 5 hrs. 30 min. 36 sec. **19.** 1 ft. 6 in. **20.** 4 dys. 23 hrs. 20 min.
21. 40 yds. **22.** 16 dwt. 8 grs. **23.** 39 tons 1432 lbs.
24. $\cdot172+$ **25.** $\cdot215$. **26.** $\cdot258\dot{3}$. **27.** $\cdot0\dot{1}5$. **28.** $\cdot8\dot{3}$.
29. $2\cdot75+$ **30.** $\cdot2291\dot{6}$. **31.** $\cdot41\dot{6}$. **32.** $\cdot418\dot{2}$. **33.** $16\cdot8595+$
34. $\cdot0464+$ **35.** $\cdot3263+$

X. Pages 12, 13.

1. $\cdot4$. **2.** $2\cdot3625$. **3.** 5. **4.** $\frac{33}{367}$. **5.** 7d. **6.** 13 cwt. **7.** $\cdot525$.
8. \$728. **9.** $\cdot0027+$ **10.** $\frac{311}{3300}$. **11.** $\frac{14}{24}$. **12.** $2\frac{7}{8}\frac{9}{2}\frac{1}{5}$.
13. $\frac{27}{288}$. **14.** $21\frac{7}{8}$ lbs. **15.** 10 rods 2 yds. $2\frac{3}{5}$ in. **16.** 10.
17. The former by $\cdot000\dot{1}$. **18.** $\frac{183}{2}\frac{315}{078}$. **19.** 4320 times.
20. $\cdot000125$. **21.** $\frac{63}{3000}$. **22.** 6. **23.** 2253·456 min. **24.** 1010.
25. 1150. **26.** $206\frac{1}{4}$ yds.

XII. Pages 16, 17.

37. $10\frac{1}{3}$. **38** $2\frac{1}{2}\frac{1}{7}$. **39.** $89\frac{1}{6}$. **40.** $\frac{405}{1072}$. **41.** $12\frac{2}{15}$.
42. $\cdot003649$. **43.** $\cdot093$. **44.** $\cdot0072$. **45.** $5\cdot15$. **46.** $51\frac{3}{35}$.
47. $\frac{512}{13125}$. **48.** 57. **49.** 36. **50.** $\cdot017$. **51.** $\cdot6891+$
52. £11. 13s. 4d. **53.** \$10,968. **54.** \$9720. **55.** \$8465.
56. 45 tons 10 cwt. **57.** 900 days. **58.** 35 gal. **59.** \$5.28.
60. $7\frac{1}{2}$ cents. **61.** $7957\frac{43}{71}$ miles. **62.** $\frac{8}{15}\frac{9}{7}$. **63.** £56. 5s.
64. 1 lb. $14\frac{10}{17}$ oz. **65.** 140 hrs. $37\frac{1}{2}$ min. **66.** $\$8.32\frac{1}{2}$.
67. 7 lbs. 13 oz. **68.** 3 tons 400 lbs. $6\frac{1}{4}$ oz. **69.** 178·855 mi.
70. \$438. **71.** 19th May. **72.** 65 days. **73.** \$720.
74. 2390 men. **75.** $\cdot979$. **76.** 24 days.

XIII. Pages 18, 19.

11. (*a*) \$35, \$49, \$56. (*b*) \$40, \$44, \$56. (*c*) \$20, \$40, \$80.
12. (*a*) A \$26, B \$40. (*b*) A \$43, B \$23. (*c*) A \$45$\frac{2}{3}$, B \$20$\frac{1}{3}$.
(*d*) A \$41$\frac{2}{3}$, B \$24$\frac{1}{3}$.
13. (*a*) A \$20, B \$10, C \$15. (*b*) A \$25.50, B \$12.75, C \$6.75.
(*c*) A \$24, B \$14, C \$7. (*d*) A \$23.75, B \$9.37$\frac{1}{2}$, C \$11.87$\frac{1}{2}$.
14. Each man \$10, each woman \$7.50, each child \$2.50.
15. Each man \$11.25, each woman \$7.50, each child, \$5.
16 \$497, \$355. **17.** \$61.25, \$78.75. **18.** \$106$\frac{2}{3}$, \$100, \$133$\frac{1}{3}$.
19. $9\frac{3}{13}$ days. **20.** 16 days. **21.** 339, 226, 113. **22.** A \$$49\frac{3}{13}$,
B \$$30\frac{10}{13}$. **23.** A \$15,600, B \$20,400, C \$24,000.
24. 12 hrs. **25.** 16 days.

XIV. Pages 20, 21.

1. 15. **2.** 16. **3.** $\frac{20}{35}\frac{14}{75}$. **4.** $\frac{42}{125}$. **5.** \$34.12$\frac{1}{2}$. **6.** 14 lbs. $2\frac{5}{16}$ oz.
7. $5\frac{3}{8}$ wks. **8.** 130 yds. 2 ft. **9.** 15 horses. **10.** 160 bushels.
11. 18 days. **12.** 8 men. **13.** 15 men. **14.** $22\frac{10}{11}$ days.
15. 219 days. **16.** $1\frac{1}{3}$ days. **17.** \$153.60. **18.** $5\frac{1}{3}$ months.
19. 160 strokes. **20.** 15 days. **21.** £2. 10s. **22.** 60 men.
23. $6\frac{9}{7}$ days. **24.** \$2064. **25.** 50 men.

XV. Pages 22, 23.

1. 12 yrs. $8\frac{1}{2}$ mos. **2.** \$287$\frac{1}{3}$. **3.** $1\frac{2}{3}$ mistakes. **4.** 200 yds.
5. 6 hrs. **6.** 52%. **7.** 35 gal. **8.** 4%. **9.** 98 clerks. **10.** \$536$\frac{2}{3}$.

XVI. Pages 23, 24.

1. $2\frac{8}{11}$ days. **2.** $3\frac{1}{13}$ hrs. **3.** 8 hrs. **4.** $12\frac{1}{2}$ hrs. **5.** $6\frac{1}{2}$ hrs.
6. $17\frac{1}{7}$ min.; 24 min. **7.** $3\frac{3}{4}$ hrs. **8.** A $19\frac{1}{8}$ dys.; B $13\frac{7}{8}$ dys.;
C $10\frac{3}{8}$ dys. **9.** A 15 dys.; B 18 dys.; C 20 dys.

XVII. Pages 24–26.

1. 45 min.; 4 hrs. **2.** $7\frac{11}{15}$ mi.; $1\frac{13}{15}$ hrs. **3.** $9\frac{1}{7}$ mi.; $1\frac{13}{15}$ hrs.
4. $2\frac{1}{2}$ hrs.; $13\frac{1}{8}$ mi. **5.** 156 mi.; 1.15 P.M. **6.** A. **7.** 12.35 P.M.;
$80\frac{7}{9}$ mi. **8.** 80 yds.; 6 yds. 2 ft. **9.** 7.10$\frac{1}{2}$ P.M. **10.** 48 mi.
11. Ratio 8 : 9. **12.** 580 yds.
13. 2 hrs. $10\frac{10}{11}$ min.; 6 hrs. $32\frac{8}{11}$ min.; 10 hrs. $54\frac{6}{11}$ min.
14. 3 hrs. $49\frac{2}{11}$ min.; 7 hrs. $5\frac{6}{11}$ min.; 9 hrs. $16\frac{4}{11}$ min.

XVII. Pages 24–26 (*continued*).

15. (*a*) 4 hrs. $5\frac{5}{11}$ min. ; 4 hrs. $38\frac{2}{11}$ min. (*b*) 10 hrs. $5\frac{5}{11}$ min. ; 10 hrs. $38\frac{2}{11}$ min. (*c*) 1 hr. $21\frac{9}{11}$ min. ; 1 hr. $54\frac{6}{11}$ min.
16. In 120 days (2 P.M. April 24th) ; 1.44 P.M. and 2.14 P.M.
17. 5.10 P.M. **18.** 12th May ; 5 P.M.

XVIII. Page 26.

1. 244·4625 francs ; $\$3.68\frac{4}{10}$. **2.** 167 fr. **3.** £11. 3s. 3d.
4. 30 cents. **5.** $56\frac{7}{10}$ lbs.

XXI. Pages 29, 30.

1. \$366.50. **2.** \$6480 ; \$4728.50. **3.** (*a*) \$156.06 ; (*b*) \$46.41 ; (*c*) \$109.37½. **4.** (*a*) \$2400 ; (*b*) \$8400 ; (*c*) \$18,000. **5.** (*a*) \$5100 ; (*b*) \$236.91 ; (*c*) 15,506.23. **6.** (*a*) \$6825 ; (*b*) \$4696.65 ; (*c*) \$4345.37.
9. 2%. **10.** \$161.10. **11.** \$260. **12.** 2035 barrels (nearly).

XXII. Pages 30, 31.

1. (*a*) \$25 ; (*b*) \$10 ; (*c*) \$487.50. **2.** (*a*) 1½ p.c. ; (*b*) 2⅓ p.c.
3. (*a*) \$2000 ; (*b*) \$10,520 ; (*c*) \$6800. **9.** \$765.62½. **10.** \$6666.66⅔.
11. \$40.84. **12.** \$30,000.

XXIII. Pages 32, 33.

A. **3.** \$177.60. **4.** 4⅜ p.c. nearly. **5.** \$17.76. **6.** \$84,000. **7.** \$3000.

B. **2.** \$20,475. **3.** \$149.45. **4.** \$1234.80. **5.** \$7·095. **6.** \$46.19. **7.** \$33.44. **8.** \$2190.69. **9.** \$994.31. **10.** \$3039.75 ; \$9394.88. **11.** \$903.60. **12.** \$1440.

XXIV. Pages 34–37.

2. (*a*) \$527 ; \$31. (*b*) \$183.60 ; \$5.10. (*c*) \$5257.50 ; \$225. (*d*) \$1168.31¼ ; \$46.50. (*e*) \$1111.50 ; \$48.00.
3. (*a*) \$875 ; \$35. (*b*) \$1920 ; \$96. (*c*) \$2066⅔ ; \$62.
4. (*a*) 79¾. (*b*) 97⅛. (*c*) 102¼. **5.** (*a*) \$875. (*b*) \$256.66⅔. (*c*) \$1733.33⅓. (*d*) £2880 4s. 2d. **6.** (*a*) \$6733⅓. (*b*) \$11,312.50. (*c*) \$6945. (*d*) £1843.
7. (*a*) 72⅛. (*b*) 88⅜. (*c*) 78. (*d*) 73⅜. **8.** (*a*) $3\frac{3}{4}$%. (*b*) $4\frac{1}{2}$%. (*c*) $3\frac{2}{3}$%. (*d*) $5\frac{1}{4}$%.
9. (*a*) $3\frac{3}{4}$%. (*b*) $3\frac{1}{4}$%. (*c*) $4\frac{2}{3}$%. (*d*) $3\frac{1}{3}$%. **10.** (*a*) the 3½ per cents. (*b*) the same. (*c*) the 3⅓ per cents. (*d*) the $3\frac{3}{4}$ per cents. (*e*) the 3⅓ per cents. **11.** (*a*) loss of \$3.33⅓. (*b*) gain of \$109.50. (*c*) loss of 75 cents. (*d*) loss of \$1.80. (*e*) gain of \$3.50.

XXIV. Pages 34–37 (*continued*).

12. \$8000. **13.** \$300. **14.** $4\frac{1}{6}$%. **15.** \$4925. **16.** $83\frac{1}{3}$.
17. £26,933. 6s. 8d. **18.** \$32.41. **19.** $7\frac{31}{117}$ p.c. **20.** \$7560.
21. $86\frac{2}{3}$ acres. **22.** £$119\frac{7}{8}$. **23.** Rates $4\frac{1}{21}$ and $4\frac{1}{31}$ p.c. ; as 155 to 147. **24.** £90. **25.** \$3104. **26.** \$$22\frac{1}{8}$ gain. **27.** \$715.
28. £222. 10s. loss. **29.** The former by $\cdot 00\frac{2}{3}$. **30.** 60%.

XXVI. Pages 39–45.

A. **1.** \$54.95. **2.** \$49.22. **3.** \$35.96. **4.** \$48.98. **5.** \$88.60.
6. \$17,520.81. **7.** \$171.69. **9.** Balance due from W. \$1.91.
10. Balance due from H. \$3.57½. **12.** Balance due to S. \$0.26.

B. **3.** \$720.64. **4.** 4th Sept. ; \$44.71 ; \$3955.29. **6.** \$343.01 (93 days). **7.** \$337.36. **11.** \$664 balance. **13.** \$658.54.
15. \$1360. **16.** \$576.49. **17.** (*c*) face of cheque \$307.03.

XXVII. Pages 46–48.

1. 70 loads. **2.** \$264. **3.** 20 tons. **4.** 7 tons 1625 lbs. **5.** $1\frac{7}{15}$ lbs.
6. 28,000 silk-worms. **7.** 9 lbs. 1 oz. 7 dwt. 12 grs. **8.** 273 lbs. 6090 grs. **9.** 1240 grs. ; $2722\frac{2}{9}$ lbs. **10.** $15\frac{9}{25}$ lbs.
11. 1 hr. 12 min. 48 sec. **12.** 6 wks. 4 dys. 9 hrs. 22 min.
13. \$9.36. **14.** 101¼ miles. **15.** 200 miles. **16.** 25·43+sec.
17. $494\frac{88}{99}$ sec. ; a little more than $\frac{1}{8}$ of a second. **18.** 631 pieces ; rem. ·033 inch. **19.** 24 ropes. **20.** 120 times ; 22 yds. ; 4 rods.
21. 7·92 in. **22.** 12 mi. 880 yds. **23.** 1788·38 yds. **24.** 21 ac. 2·28 sq. chains. **25.** 212·16 sq. rods. **26.** 4·98 acres.
27. 2 acres. **28.** 934·12 sq. yds. **29.** 41 ft. $2\frac{11}{25}$ in. **30.** $67\frac{2}{3}$ loads.
31. 6·2321 gal. ; 997·136 oz. **32.** 3750 lbs. **33.** 41,250,000 tons.
34. 5624·64 oz. **35.** \$2637.43. **36.** 512,000 cub. ft.
37. 599 miles. **38.** 1694 mi. 560 yds.

XXVIII. Pages 49–52.

1. 618·334 Kg. **2.** 1·20 f.. **3.** 125 l. **4.** 113·50 fr. **5.** 14·25 Km.
6. 1020·276 Km. **7.** 92 francs. **8.** 10 m. **9.** 24,000 Kg.
10. 2323·2 Hl. **11.** 27,000 l. **12.** 1 Kg. **13.** 55·2 metric tons.
14. 45·2088 Kg. **15.** 5670 francs. **16.** 10,070 l. ; 10,070 Kg.
17. 3 francs. **18.** 62·25 Kg.

XXVIII. Pages 49–52 (*continued*).

19. 1·093 yds.; 2·539 cm.; ·914 m.; 1·609 Km. **20.** 1550·016 sq. in.; 6·451 sq. cm.; ·836 sq. m.; ·404 Ha. **21.** ·061 cu. in; ·275 cord; 16·386 cu. cm.; ·764 cu. m. **22.** ·568 l.; ·045 Hl.; 36·36̇ l. **23.** ·015 grains; ·064 g.; 28·349 g.; 31·103 g.; ·453 Kg. **24.** 19 mi. 19·6 rods. **25.** 2·4068 cu. m. **26.** 285·71 l. **27.** 8·178 cu. m. **28.** 9 dwt. 19·343 grs. **29.** 2 cu. ft. 1502 cu. in. **30.** ·189 in. **31.** $5\frac{5}{12}$ lbs.; 2·457 Kg. **32.** 3 hrs. 51·7 min. **33.** 15·16+Kg. **35.** ·03937 inches; $3280\frac{5}{6}$ ft. **36.** 61·02 cu. in. **37.** 567 grains. **38.** 66·93 lbs. Troy. **39.** 5 l. **40.** 484 cm. (nearly). **41.** ·474 l.

XXIX. Pages 53–55.

A. **1.** 1681. **2.** 1156. **3.** 855625. **4.** $39\frac{1}{16}$. **5.** $592\frac{1}{8}$. **6.** 2524·0576. **7.** 29791. **8.** 1·442897. **9.** 1015·075125. **10.** $\frac{6}{8}\frac{8}{8}\frac{3}{5}\frac{2}{0}$. **11.** $\frac{10851569}{21384719}$. **12.** 31255·875. **13.** 17596287801. **14.** $21\frac{204}{2401}$. **15.** 118587876497. **16.** $\frac{3125}{7776}$. **17.** $\frac{241375099}{837661121}$. **18.** 781. **19.** ·035012791. **20.** 92100·25. **21.** 525. **22.** 163210432. **23.** $1580\frac{1101}{10850}$. **24.** 14·885593̇. **25.** 53·841087. **26.** ·118955463.

B **1.** 24. **2.** 41. **3.** 34. **4.** 69. **5.** 137. **6.** 428. **7.** 102. **8.** 516. **9.** 1·262. **10.** 15·3. **11.** ·0231. **12.** 32·768. **13.** $6\frac{1}{2}$. **14.** $6\frac{1}{4}$. **15.** $24\frac{1}{3}$. **16.** $10\frac{3}{4}$. **17.** $\frac{1}{2}$. **18.** $\frac{4}{5}$. **19.** $\frac{8}{27}$. **20.** $\frac{1}{2}$.

21. 1·414. **22.** 2·449. **23.** 28·213. **24.** 9·022. **25.** 2·091. **26.** 2·886. **27.** ·894. **28.** ·031. **29.** 22·992. **30.** 37·21. **31.** 15·368. **32.** 1·038.

33. 15. **34.** 12. **35.** 42. **36.** 9·539. **37.** ·16̇. **38.** ·45. **39.** ·03. **40.** ·24.

41. 327 yards. **42.** 126 soldiers. **43.** 76 rods. **44.** 205 rods. **45.** 72 yards. **46.** 160 rods; 80 rods. **47.** 3·6 inches. **48.** 6·928 ft. **49.** 20 ft. **50.** 103·614+ft. **51.** 75·816+ft.

C. **1.** 13. **2.** 23. **3.** 27. **4.** 79. **5.** 341. **6.** 192. **7.** 512. **8.** 611. **9.** 1·07. **10.** 30·1. **11.** ·067. **12.** 6·16. **13.** $\frac{3}{4}$. **14.** $2\frac{1}{2}$. **15.** $3\frac{1}{3}$. **16.** $2\frac{3}{5}$. **17.** $17\frac{3}{7}$.

18. 4·179. **19.** 4·762. **20.** ·482. **21.** ·149. **22.** 3·264. **23.** $1\frac{53}{140}$. **24.** 539·8 nearly. **25.** 76 inches. **26.** 1 cub. ft. **27.** (*a*) 15 min. (*b*) 12 yds **28.** (*a*) 62·5 lbs. (*b*) 6 in.

XXX. Pages 56-60.

A. **1.** 70 feet. **2.** 37 rods. **3.** 6 links. **4.** $4\frac{1}{2}$ yards. **5.** 7 ft. 6 in. **6.** \$93.09. **7.** \$75.35. **8.** 975 links.

B. **1.** (*a*) 350 sq. yards. (*b*) 89·3 sq. yds. (*c*) 8 ac. 1 rod. (*d*) 13·69 sq. in. (*e*) $150\sqrt{14}$ sq. ft. (*f*) 70·1478 sq. yds. **2.** 14 yds. **3.** 768 rods. **4.** \$0.66. **5.** 14 ac. 4440 sq. yds. **6.** 582·06 yds. (nearly). **7.** 25 chains. **8.** 150, 200, 250 yards; 45,000 sq. yards.

C. **4.** $6\frac{3}{4}$ ac. **5.** 12,480 sq. yds. **6.** 4 ac. 2 roods. **7.** 1 ac. 134 rods. **8.** 5 ac. $155\frac{85}{250}$ rods. **9.** ·248 ac. **10.** 7 chains 70 links.

D. **3.** (*a*) $5028\frac{4}{7}$ sq. ft. (*b*) $2011\frac{3}{7}$ ac. (*c*) $103\frac{51}{56}$ sq. ft. **4.** (*a*) $3181\frac{9}{11}$ sq. yds. (*b*) 8 sq. ft. nearly. **5.** (*a*) 3·98 ft. (*b*) 27·7 yds. **6.** $114\frac{3}{7}$ sq. ft. **7.** 336 times. **8.** 135·96 yds.; 146·86 yds.; 5·44 yds. **9.** 50·92 ac. **10.** $640\sqrt{\pi} = 1134·08$ yards. **11.** $64\frac{7}{7}\frac{2}{7}$ sq. rods. **12.** 1145·45 sq. ft.; 692·8 sq. ft.; 900 sq. ft. **13.** 12,432 revolutions. **14.** 968 yds. **15.** 15·093 in. **16.** 39·25 yards.

XXXI. Pages 61-68.

A. **4.** (*a*) $9\frac{3}{8}$ sq. ft. (*b*) $204\frac{1}{8}$ sq. ft. (*c*) $234\frac{3}{8}$ sq. ft. (*d*) $155\frac{1}{2}\frac{1}{4}$ sq. ft. (*e*) $630\frac{3}{8}$ sq. ft. **5.** (*a*) $8\frac{1}{3}$ sq. ft. (*b*) $64\frac{7}{3}\frac{7}{6}$ sq. ft. (*c*) $176\frac{17}{24}$ sq. ft. **6.** (*a*) $(96+4\sqrt{2})=101·656$ sq. in. (*b*) $9\frac{1}{2}$ sq. ft. nearly. **7.** (*a*) $1\frac{1}{7}$ sq. ft. (*b*) $21\frac{3}{1}\frac{3}{4}$ sq. ft. (*c*) $10\frac{17}{28}$ sq. ft. **8.** $67\frac{1}{2}$ sq. ft. **9.** $16\pi = 50\frac{2}{7}$ lbs. **10.** 10 ft. 9 in.; 99,846 sq. in. **11.** 2·45 in. per min. **12.** $100\sqrt{2} = 141·4$ ft. **13.** 5 cu. ft. 408 cu. in. **14.** 421 cu. ft. 1512 cu. in. **15.** 10·032 cu. ft.; 7·392 cu. ft.

B. **3.** 4 sq. feet. **4.** $4\sqrt{3} = 6·928$ sq. in. **5.** $37\frac{5}{7}$ sq. in. **6.** $122\frac{4}{7}$ sq. in. **7.** 13 in.; $204\frac{3}{7}$ sq. in. **8.** $32\frac{1}{1}\frac{3}{6}$ cu. ft. **9.** 2 cu. yds. 24·54 ft.; 12 sq. yds. 7·846 ft. **10.** 1 cu. yd. 9·083 ft.; 9 sq. yds. 5·59 ft. **11.** $169\frac{3}{7}$ cu. m. **12.** $38\frac{1}{2}$ sq. ft. **13.** £16. 13s. 4d. **14.** $53\frac{1}{8}$ sq. yds. **15.** $6\frac{1}{4}$ ft. **16.** $77\frac{3}{2}\frac{1}{2}$ cu. ft. **17.** $11\frac{2}{5}\frac{0}{1}\frac{9}{2}$ ft. **18.** 93391360 cu. ft.

C. **3.** $201\frac{1}{7}$ sq. in. **4.** 201062400 sq. mi. **5.** $523\frac{1}{2}\frac{7}{1}$ cu. in. **6.** $381\frac{9}{7}$ cu. in. **7.** $14\frac{1}{7}$ cu. in. **8.** $245\frac{1}{2}\frac{6}{8}$ lbs. **9.** 11 : 21. **10.** 1403 lbs. nearly. **11.** $\sqrt[3]{\frac{3}{4\pi}}$ to 1 or 31 to 50. **12.** $4\frac{4}{7}\frac{4}{1}$ cu. ft. **13.** 1 : 6. **14.** $709\frac{1}{2}$ lbs.

XXXI. Pages 61–68 (*continued*).

D. 3. 441 cub ft. 4. 40 sq. ft. ; 53 sq. ft.
5. $12\frac{1}{7}$ sq. in. 6. $122\frac{1}{7}$ sq. in. 7. £858. 15s. 3d. (nearly).
8. 175 sq. in. 9. $257\frac{5}{7}$ sq. yds. 10. $1186\frac{20}{33}$ cu. ft.
11. 39 tons $1880\frac{20}{21}$ lbs. 12. 11 sq. ft. 13. $6\frac{13}{15}$ times.

XXXIII. Pages 73–75.

A. 1. $\cdot\dot{4}2857\dot{1}$. 2. \$7295.43. 3. \$73.56. 4. 10%. 5. 15 rupees.

B. 1. $\frac{1}{2}$. 2. 2942·2775 sq. yards. 3. 8 days. 4. $4\frac{7}{12}$ p.c.
5. The same, \$605.28.

C. 1. $\frac{1}{3}$; $1\frac{6}{13}$ $(1\frac{7}{13} - \frac{1}{13})$. 2. $6\frac{2}{5}$ min. 3. A £22. 10s. ; B £37. 10s. ;
C £45. 4. 2364 mi. $1486\frac{1}{2}$ yds. 5. $117\frac{1}{2}$.

D. 1. ·0021+. 2. $27\frac{3}{11}$ minutes past 2.
3. St. Petersburg, 2 hrs. 1 min. 20 sec. P.M.
Berlin, 0 hrs. 53 min. 36 sec. P.M.
Dublin, 11 hrs. 34 min. 56 sec. A.M.
New York, 7 hrs. 4 min. 8 sec. A.M.
4. $\$50{,}350\frac{5}{8}$. 5. 24,768 tiles.

E. 1. 16·1. 2. $\$163.87\frac{1}{2}$. 3. \$12.60 gain. 4. $41\frac{1}{7}$ ft.
5. 123·888 sq. ft. ; 7·87 ft.

F. 1. 4·23+ oz. ; 58·293 m. 2. $11\frac{1}{9}$ p.c. 3. $\$0.88\frac{3}{4}$.
4. \$240 ; \$6720. 5. $64\pi = 201\frac{1}{7}$ sq. miles.

G. 1. 4·38+ 2. 27.27 l. 3. 14,479,674 gal. 4. 2 23608 lbs.
5. 2513·28 cu. ft.

XXIX. Pages 81–101.

1. 6; 6; 1; 1; $\bar{6}$; $\bar{1}$; $\bar{4}$; $\bar{1}$. **2.** 4; $\bar{3}$; 4; $\bar{6}$; $\bar{5}$; $\bar{3}$; $\bar{1}$. **3.** 2; 3; $\bar{2}$; 2; 5; $\bar{1}$; $\bar{2}$; 1; 5; 6. **4.** ·1; 2; $\bar{1}$; $\bar{2}$; 1; 2; $\bar{1}$; $\bar{2}$; 1; $\bar{1}$. **5.** 1000000; 1000000; 10; 10; ·000001; ·1; ·0001; ·1; 10000; ·001; 10000; ·000001; ·00001; ·001; ·1; 100; 1000; ·01; 100; ·00001; ·1; ·01; 10; 100000; ·000001; 10; 100; ·1; ·01; 10; 100; ·1; ·01; 10; ·1. **6.** 100; 100; 10000; 100; ·000001; 100; ·01; 10; 10; 100; 1; 1. **7.** ·69897; ·60206; 1·30103; 1·39794; 2·30103; 1·50515; 50515; ·90309; 2·39794; 1·69897; 2·70927; 1·70927; ·70927; 2·09691; ·09691; 2·60206. **8.** 1; 3; $\bar{1}$; $\bar{4}$; $\bar{4}$; $\bar{3}$; 0; 0; 0; 3; $\bar{6}$; $\bar{1}$. **9, 10, 11, 12** and **13.** The answers to these exercises are easily verified by ordinary arithmetic. **14.** ·594171; ·678063; ·992421; ·771073; ·680336; ·832509; ·874772; ·724276; ·867467; ·290480; ·003029. **15.** ·671265; ·671265; ·671265; ·944483; ·944483; ·944483; ·944483; ·944483; ·944483; ·845098; ·845098; ·845098; ·845098; ·724276; ·938019; ·979548; ·688064; ·976902; ·204120; ·000000; ·000000; ·000000; ·000000; ·000000; ·000000; ·000000; ·733037; ·828015; ·992156. **16.** 3·671265; 1·671265; $\bar{2}$·671265; 3·944483; 2·944483; 1·944483; 0·944483; $\bar{1}$·944483; $\bar{2}$·944483; $\bar{3}$·944483; 3·845098; 2·845098; 1·845098; 0·845098; 0·724276; $\bar{2}$·938019; 5·979548; 1·688064; 0·976902; $\bar{3}$·204120; 3·000000; 2·000000; 1·000000; 0·000000; $\bar{1}$·000000; $\bar{2}$·000000; $\bar{3}$·000000; $\bar{4}$·000000; $\bar{1}$·733037; $\bar{4}$·828015; $\bar{2}$·992156. **17.** 1·452859; $\bar{1}$·979821; $\bar{1}$·681241; 0·755417; 0·895533; 1·988291; 2·632153; $\bar{3}$·834548; 1·776047; $\bar{2}$.776846; $\bar{1}$·771367; $\bar{3}$·765669; $\bar{2}$·792392; $\bar{2}$·845098; $\bar{1}$·903090; 1·979639; 2·986010; 4·900586; 5·166726; 5·985426. **18.** ·1557; 2605; 3662; 5017; 6157; 7469; 8000; 8666. **19.** 580000; 67610; 8062; 935·1; 15·46; 2·142; ·2758; ·03367; ·003634; ·0003958; ·00004316. **20.** ·139753; ·141356; ·424519; ·596461; ·633415; ·755015; ·755015; ·755015; ·755015; ·755015; ·970635; ·897377; ·776693. **21.** 0·566685; 1·656883; $\bar{3}$·832094; $\bar{2}$·902538; 8·981490; 2·984410; 1·068869; $\bar{4}$·774463; 1·902255; 3;440606. **22.** ·553757; ·834092; ·900893; ·903034, ·936119; ·948363; ·979284; ·986226; ·988420; ·999440. **23.** ·554443; ·680512; ·769877; ·770656; ·791126; ·796217; ·896909; ·951280; ·960871; ·972150. **24.** 5·566168; 4·734984; 5·885839; 5·953631; $\bar{2}$·804695; $\bar{3}$·777405; $\bar{4}$·566141; $\bar{4}$·744723; 0·001649; 0·000002. **25.** 136608; 129153 or 5; 161620; 211653 or 6; 286278 or 9; 369734 or 5; 477421; 615193 or 4; 776466 or 7; 796587 or 9. **26.** 1046·83;

XXIX. Pages 101-108 (*continued*).

128·785; 16·2551; 2·09457; ·270464; ·0349309; ·0045115; ·000582684; ·0000752562; ·0000097197. **27.** 1562·33; 140·699; ·00188217; ·0320476; ·000160756; 263442; 179·722; 1·7747; 3·43356; 90023·2. **28.** 1·24544; ·925608; 1018·58; ·0486318; 1·99261; 15·3311; ·454246; ·00000138603; 85·8694; ·0116455. **29.** ·90909; ·0681105; ·0102691; ·000266328; 100; 10000; 271·54; 101·743; ·00000179602; 1·79602. **30.** ·68421; ·776567; ·254892; ·804446; ·000268455; ·7293; 723·335; ·836177; ·796747; 8·61764. **31.** 1·46154; 1·2877; 3·92325; 1·24309; 3725; 1·37111; ·00138247; 1·19588; 1·2551; ·116041. **32.** ·00342649; ·0491818; ·217178; 1·00045. **33.** 387·383; ·00636007; 454·117; ·000470505; 12·9496; 5·68826; ·0383225; 9·12956; ·0000000000000428144; 1·39433. **34.** 966; 91·3; 5·33; ·619; ·0411; 93; 3·7; 5·5; 2·0808; 8·3107; ·984129; ·233894; 1·27973; ·564185; 1·00956; 2·01546; ·615587; 1·03606; ·9564; 1·01509. **35.** 5·09147; 10·405; ·683938; ·758415; ·00000000444493. **36.** See answers in proper places. **37.** $719.65; $772.64; $2621.20; $1712.71. **38.** 18·502 acres; 37·8729 acres; 164·208 acres; 22·719 acres; 13·7007 acres; 21·1366 acres; 6·88086 acres; 3·25469 acres. **39.** 9 volumes would be needed, but the last volume would have 4 blank pages, the last page 3 blank lines, and the last line 50 blank spaces.